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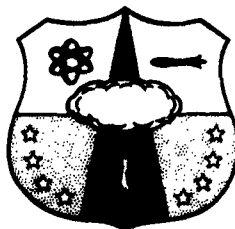
SPINNING UNGUIDED ROCKET TRAJECTORY  
DIGITAL COMPUTER PROGRAM (SPURT)

by

Allen D. Dayton  
Lt USAF

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Test Directorate  
AIR FORCE SPECIAL WEAPONS CENTER  
Air Force Systems Command  
Kirtland Air Force Base  
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## FOREWORD

The author wishes to acknowledge the assistance of Mr. Carl S. Christensen, who did much of the original logic and programing of SPURT. Mr. Christensen, now with the Aerospace Corporation, Los Angeles, California, was formerly a lieutenant at the Air Force Special Weapons Center.

ABSTRACT

SPURT is a five-degree-of-freedom trajectory digital computer program for spinning unguided space probe vehicles. The program was written for the Control Data Corporation, 1604 digital computer.

SPURT will compute trajectories for a vehicle up to a maximum of ten stages and has provision for computing the trajectories of the separated stages.


This generalized program computes the trajectory over an oblate spheroidal, rotating Earth with atmosphere, in a geocentric rectangular coordinate system. All input and output data are in geodetic coordinates.


Coasting flight trajectories are computed in two subroutines. The first is a Keplerian solution, which also computes orbital elements and "look angles" for various tracking stations. The second uses three-degree-of-freedom point mass equations solved by numerical integration.

The program will prepare two special output tapes. One is used in plotting output data and the other is used to prepare a special tape for the Atlantic Missile Range.

PUBLICATION REVIEW

This report has been reviewed and is approved.

  
ALBERT L. HALEY  
Colonel USAF  
Director, Test Directorate

  
JOHN J. DISHUCK  
Colonel USAF  
DCS/Plans & Operations

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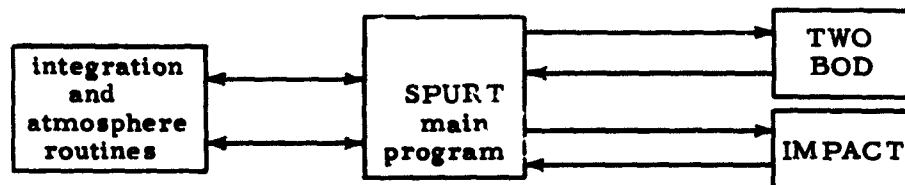
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# 1. INTRODUCTION.

The Spinning Unguided Rocket Trajectory (SPURT) digital computer program is designed to provide a generalized computer program for calculating trajectories of spinning unguided space probe vehicles such as the SLV-1B. Many trajectory programs are available<sup>1-5</sup> but none meet the demands required by the Space Vehicle Branch (SWTTS) of AFSWC. The program is written in a mixture of FORTRAN and CODAP for use on the CDC-1604 computer and utilizes subroutines of the CO-OP library. The program is designed to handle up to a ten stage vehicle for both powered and unpowered flight with provisions to calculate the trajectories of the separated "expended" stages.

The main program computes the powered portion of the trajectory and controls entry into the various subroutines. All unpowered flight trajectories are computed in the two subroutines "TWO BOD" and "IMPACT." Both of these subroutines have the capability to calculate the trajectory of all the separated stages and of the payload. TWO BOD is a Keplerian trajectory program with provisions for calculating look angles of various tracking stations, while IMPACT integrates the equation of a point mass with drag.



The main program uses a five-degree-of-freedom, three-dimensional system with the sixth degree constrained by a table of spin rates that are read into the program. An oblate rotating earth and associated gravitational potential, standard 1962 atmosphere, and altitude-dependent wind provisions are also incorporated into the program. The position vector is calculated in a rotating Earth-centered coordinate system, while the angular positions are calculated in a launch-centered coordinate system.

The procedure includes provisions for coasting periods between stages, which are terminated by time. Thrust is computed from thrust vs. time tables and corrected for atmosphere back pressure.

Aerodynamic forces and moments about the center of mass are interpolated from a table of Mach number dependent coefficients.

Computations are carried out using either the Adams or the Runge-Kutta method of numerical integration. Both methods can be used in either fixed or variable step size-mode.

The program has the provision for writing two special output tapes. One, a plot tape, is designed to be used with a special plot program for use on the AFSWC plotter and can plot any output variable against any other output variable. The second, a BATT tape, is used with the BATT program to prepare magnetic tapes to meet specified Atlantic Missile Range formats.

The program running time is approximately 3 to 4 minutes for a four-stage vehicle similar to the SLV-1B. The integration and atmosphere routines are compiled separately in octal locations 6000 to 7514. The rest of the program is compiled in fixed binary mode starting at octal location 10050. The two parts are then put together to be read into the computer. This method has the advantage that the integration and atmosphere routines are not recompiled every time a change is made in the rest of the program.

The program is stored in core according to the following octal addresses.

6000 - 7142	Integration Routine	
7300 - 7514	Atmosphere Routine	
7660 - 10026	SQRTF, EXPF, & LOGF Routines	
10050 - 34102	SPURT Routine (main program)	
34103 - 34262	SETTAB Routine	} subroutines
34263 - 34350	ECLOCK Routine	
34351 - 34364	SCLOCK Routine	
34365 - 56750	TWO BOD Routine	
56751 - 61516	IMPACT Routine	
61517 - 61615	GEODED Routine	
61616 - 62006	ROTATE Routine	
62013 - 67420	LIBRARY Routines	
70037 - 71343	COMMON	

2. EQUATIONS.SYMBOLS

		<u>Dimensions</u>
A	Axial moment of inertial	ft <sup>2</sup> - slug
A <sub>E</sub>	EXIT area of nozzle	in <sup>2</sup>
A <sub>w</sub>	Azimuth angle* of the wind	deg
A <sub>x</sub>	Azimuth angle of the X <sub>L</sub> axis	deg
a <sub>E</sub>	Equatorial radius of the Earth	ft
B	Longitudinal moment of inertia	ft <sup>2</sup> - slug
C <sub>A</sub>	$1/\sqrt{1-(2f-f^2)\sin^2\theta_G}$ (Ref 6)	ft
C <sub>D</sub>	Drag coefficient	None
C <sub>DB</sub>	Powered flight drag coefficient	None
C <sub>DC</sub>	Coasting flight drag coefficient	None
C <sub>N<sub>a</sub></sub>	$\left(\frac{\partial C_N}{\partial \alpha}\right)$ Normal force coefficient with respect to angle of attack	per radian
C <sub>M<sub>a</sub></sub>	Moment coefficient with respect to angle of attack	None
C <sub>P</sub>	Center of pressure of the missile**	ft
D	Diameter of the missile	ft
D	Total drag on the missile	lb <sub>f</sub>
d	Derivative of a variable	None
d	Reference length	ft
e <sub>E</sub>	Eccentricity of the Earth	None
$\vec{F}$	Total force vector on the missile	lb <sub>f</sub>
f	Flattening of the Earth	None

\* All azimuth angles are measured clockwise from true north.

\*\* Measured from the tail.

		<u>Dimensions</u>
$G$	Missile center of gravity*	ft
$g_E$	Acceleration of gravity constant	$32.174 \frac{\text{lb}_m - \text{ft}}{\text{lb}_f - \text{sec}^2}$
$G_i$	Initial center of gravity of the missile*	ft
$\vec{G}_M$	Overturning moment of the missile	ft-lbs
$GM$	Gravitational constant of the Earth	$\text{ft}^3/\text{sec}^2$
$G_P$	Center of gravity of the propellant*	ft
$G_x, G_y, G_z$	Gravitational attraction components	$\text{ft}/\text{sec}^2$
$\vec{H}$	Angular momentum vector	lb-ft-sec
$H_G$	Geodetic altitude	ft
$i, j, k$	Unit vectors along X, Y, Z	None
$J$	Earth oblateness constant	None
$K$	Earth oblateness constant = $f(J)$	$\text{ft}^2$
$K_{AP}$	Propellant axial radius of gyration	ft
$K_{BP}$	Propellant transverse radius of gyration	ft
$M$	Mass of the missile	slugs
$M_i$	Initial mass of the missile	slugs
$M_P$	Mass of the propellant	slugs
$M. N.$	Mach number of the missile	None
$N$	Spin rate of the vehicle	rad/sec
$N, E$	North and east directions at the launch site	None
$P_a$	Pressure of the atmosphere	$\text{lb}_f/\text{in}^2$
$P_{aT}$	Pressure of the atmosphere at which the thrust is measured	$\text{lb}_f/\text{in}^2$
$P_E$	Constant in the $R_E$ equation	None
$\vec{R}$	Position vector of the missile	ft

\* Measured from the tail.

		<u>Dimensions</u>
$R_E$	Radius of the Earth	ft
$S$	Reference area of the missile	ft <sup>2</sup>
$S_{\Delta}$	$C (1 - f^2)$ (Ref 6)	ft
$\vec{T}$	Thrust vector of the missile	lb <sub>f</sub>
$t$	Time - independent variable	sec
$T_{AS}$	Reference temperature of the atmosphere	°K
$T_A$	Temperature of the atmosphere	°K
$T_{Tt}$	Thrust known for input data	lb <sub>f</sub>
$T_{TV}$	Vacuum thrust of the missile	lb <sub>f</sub>
$V$	Velocity of the missile	ft/sec
$V_{SD}$	Speed of sound	ft/sec
$V_{SDS}$	Reference speed of sound	ft/sec
$V_x, V_y, V_z$	Velocity components along X, Y, Z	ft/sec
$V_{wx}, V_{wy}, V_{wz}$	Wind velocity components along X, Y, Z	ft/sec
$V_1, V_2$	Velocity components along axis 1 & 2	ft/sec
$X, Y, Z$	Geocentric coordinate system	ft
$X_L, Y_L, Z_L$	Launch coordinate system	ft
$X_1, X_2, X_3$	Missile nonrotating coordinate system	None
$X_1, X_2', X_3'$	Missile rotating coordinate system	None

## GREEK LETTERS

$\alpha$	Angle between east and $X_L$ axis	deg
$\delta_T$	Thrust misalignment angle	rad
$\theta$	Euler angle as shown in figure 2	deg
$\theta_C$	Geocentric latitude	deg

		<u>Dimensions</u>
$\theta_G$	Geodetic latitude	deg
$\rho$	Density of the atmosphere	slugs/ft <sup>3</sup>
$\Sigma$	Summation sign	None
$\varphi$	Euler angle as shown in figure 2	deg
$\varphi_A$	Angle at which the thrust misalignment acts in the $X_2, X_3$ plane	rad
$\varphi_T$	Angle at which the thrust misalignment acts in the $X_2, X_3$ plane	rad
$\vec{\Omega}$	Angular velocity of missile in the $X_1, X_2, X_3$ coordinate system	rad/sec
$\vec{\omega}$	Angular velocity of the missile in the $X_1, X_2, X_3$ coordinate system	rad/sec
$\vec{\omega}_E$	$= \omega_E \vec{k}$ = Angular velocity of the Earth	rad/sec

SUBSCRIPTREFERS TO

C	Geocentric
E	The Earth
G	Geodetic
i	Initial
L	Launch system
P	Propellant
S	Reference conditions
T	Thrust
V	Vacuum
X	$X_L$ - Axis
x, y, z -	X, Y, Z Coordinate system
W	Wind
1, 2, 3	$X_1, X_2, X_3$ Coordinate system
$\alpha$	Angle of attack

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An arrow over a variable indicates that the variable is a vector.

A dot over a variable indicates the time derivative of that variable.

a. Linear equations of motion.

(1) Coordinate systems.

(a) Earth-centered coordinate system (X, Y, Z).

A right-handed, Earth-centered, Cartesian coordinate system is used. The X, Y, and Z axes, shown in figure 1 are oriented as follows:

The X axis is the node of the equatorial plane and the plane containing the Z axis and Greenwich Meridian.

The Y axis also lies in the equatorial plane and is at right angles to the X and Z axes.

The Z axis lies along the spin axis of the Earth and is at right angles to the X and Y axes.

(b) Launch coordinate system ( $X_L$ ,  $Y_L$ ,  $Z_L$ ).

A right-hand coordinate system with the  $X_L$ ,  $Y_L$  plane tangent to an oblate Earth at the launch point is used. The  $X_L$ ,  $Y_L$ ,  $Z_L$  axes are oriented as follows:

The  $X_L$  axis is in the direction of the initial launch azimuth.

The  $Y_L$  axis is counterclockwise  $90^\circ$  from the  $X_L$  axis.

The  $Z_L$  axis is positive along the local geodetic vertical.

(2) Development.

(a) Newton second law for a rotating coordinate system.

The fundamental equation of motion for a particle moving in a rotating coordinate system such as the Earth is

$$\frac{d^2 \vec{R}}{dt^2} = \frac{\Sigma \vec{F}}{M} + 2 \left( \frac{d\vec{R}}{dt} \times \vec{\omega}_E \right) - \vec{\omega}_E \times (\vec{\omega}_E \times \vec{R}) \quad (1)$$

(Reference 2)



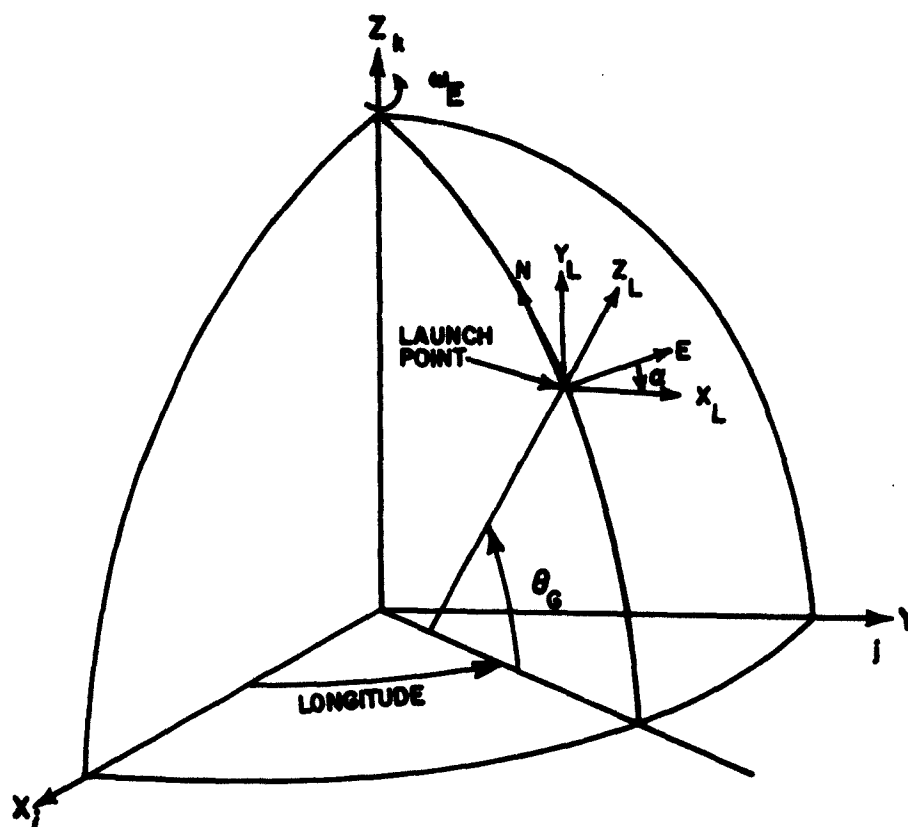


Figure 1. Coordinate systems for trajectory equations.

where  $\vec{R}$  is the vector distance from the origin of the rotating X-Y-Z coordinate system to the particle,

( $\vec{R} = \vec{i}X + \vec{j}Y + \vec{k}Z$ ).  $\frac{d\vec{R}}{dt}$  and  $\frac{d^2\vec{R}}{dt^2}$  are the velocity and acceleration,

respectively, of the particle measured with respect to the rotating axes,

$$\frac{d\vec{R}}{dt} = \vec{i}\dot{X} + \vec{j}\dot{Y} + \vec{k}\dot{Z} \quad \text{and} \quad \frac{d^2\vec{R}}{dt^2} = \vec{i}\ddot{X} + \vec{j}\ddot{Y} + \vec{k}\ddot{Z}. \quad (2)$$

$\vec{\omega}_E$  is the angular velocity of the Earth (axis system) and is along the Z axis, ( $\vec{\omega} = \vec{k}\omega_E$ ).  $\Sigma\vec{F}$  is the vector sum of the forces acting on the particle: thrust, drag, and gravity, ( $\Sigma\vec{F} = \vec{i}\Sigma F_x + \vec{j}\Sigma F_y + \vec{k}\Sigma F_z$ ). M is the total instantaneous mass of the particle.  $2\left(\frac{d\vec{R}}{dt} \times \vec{\omega}_E\right)$  is the coriolis pseudo-acceleration of the axis system due to the rotation of the Earth;  $-\vec{\omega}_E \times (\vec{\omega}_E \times \vec{R})$  is the centrifugal pseudo-acceleration of the axis system due to the rotation of the Earth.

$$\frac{d\vec{R}}{dt} \times \vec{\omega}_E = \omega_E \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{dX}{dt} & \frac{dY}{dt} & \frac{dZ}{dt} \\ 0 & 0 & 1 \end{vmatrix}$$

$$2\left(\frac{d\vec{R}}{dt} \times \vec{\omega}_E\right) = 2\omega_E \left(\vec{i}\frac{dY}{dt} - \vec{j}\frac{dX}{dt}\right)$$

$$(\omega_E \times \vec{R}) = \omega_E \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ X & Y & Z \end{vmatrix} = \omega_E (-\vec{i}Y + \vec{j}X)$$

$$\vec{\omega}_E \times (\vec{\omega}_E \times \vec{R}) = \omega_E^2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ -Y & X & 0 \end{vmatrix} = \omega_E^2 (-\vec{i}X - \vec{j}Y)$$

(3)

From equations 2 and 3 the components of equation 1 can be expressed:

$$\begin{aligned}\frac{d^2X}{dt^2} &= \frac{\Sigma F_x}{M} + 2\omega_E \frac{dY}{dt} + \omega_E^2 X \\ \frac{d^2Y}{dt^2} &= \frac{\Sigma F_y}{M} - 2\omega_E \frac{dX}{dt} + \omega_E^2 Y \\ \frac{d^2Z}{dt^2} &= \frac{\Sigma F_z}{M}\end{aligned}\tag{4}$$

The program assumes that there are three forces acting on a rocket vehicle: thrust, drag, and gravity. Thrust is assumed to act along the longitudinal axis of the rocket vehicle. Drag is assumed to act opposite the velocity vector. Gravity is assumed to be directed toward the center of mass of an oblate Earth.

(b) Gravitational attraction equations.

The equations of gravitational attraction are obtained from reference 8, and converted to a Cartesian coordinate system. They are as follows:

$$\begin{aligned}G_x &= -GM \frac{X}{R^3} \left[ 1 + \frac{3K}{R^2} - 15 \frac{KZ^2}{R^4} \right] \\ G_y &= -GM \frac{Y}{R^3} \left[ 1 + \frac{3K}{R^2} - 15 \frac{KZ^2}{R^4} \right] \\ G_z &= -GM \frac{Z}{R^3} \left[ 1 + \frac{9K}{R^2} - 15 \frac{KZ^2}{R^4} \right]\end{aligned}\tag{5}$$

Finally, the equations for total acceleration components along X, Y, and Z axes can be written as follows:

$$\begin{aligned}
 \ddot{X} &= \frac{T_x}{M} - \frac{D}{M} \frac{\dot{X}}{R} - G_x + 2\omega_E \dot{Y} + \omega_E^2 X \\
 \ddot{Y} &= \frac{T_y}{M} - \frac{D}{M} \frac{\dot{Y}}{R} - G_y - 2\omega_E \dot{X} + \omega_E^2 Y \\
 \ddot{Z} &= \frac{T_z}{M} - \frac{D}{M} \frac{\dot{Z}}{R} - G_z
 \end{aligned} \tag{6}$$

where

$$\begin{aligned}
 \dot{R} &= (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2} \\
 \ddot{R} &= (\ddot{X}^2 + \ddot{Y}^2 + \ddot{Z}^2)^{1/2}
 \end{aligned} \tag{7}$$

(c) Thrust and mass.

The mass  $M$  of the rocket vehicle and the thrust  $T_{Tt}$  at air pressure  $P_{at}$  are assumed to be known functions of time. In general these functions will be nonlinear and discontinuous. The thrust  $T$  corresponding to air pressure  $P_a$  may be computed from the equation

$$T = T_{Tt} + (P_{at} - P_a)A_e \tag{8}$$

where  $A_e$  is the nozzle exit area. ( $A_e$  is measured in square inches; therefore,  $P_{at}$  and  $P_a$  are absolute pressures in  $\text{lb/in}^2$ .)

In this trajectory-computation program, the thrust function is evaluated by table look-up and interpolation. Since this function is discontinuous, a set of tables is needed for each stage of the rocket vehicle. The tables in the working storage are changed each time a stage is dropped.

The thrust function  $T_{Tt}$  vs.  $t$  is given, but the mass function  $M$  vs.  $t$  is computed by

$$M = M_i - M_{Pi} \frac{\int_{t_i}^t T_{Tv} dt}{\int_{t_i}^{t_b} T_{Tv} dt} \quad (9)$$

where  $T_{Tv} = T_{Tt} + P_{at} A_e$  (10)

Subscript i indicates values at ignition, b denotes values at burnout, and  $M_P$  is the mass of the propellant. For clarity, subscript n denoting the configuration number has been omitted from the subscripted symbols.

(d) Aerodynamic drag.

The aerodynamic drag may be computed from the equation

$$D = \frac{1}{2} \rho S V^2 C_D (M.N.) \quad (11)$$

where

$\rho$  = Density of the air about the rocket vehicle, slugs/  
ft<sup>3</sup>

$C_D (M.N.)$  = Drag coefficient, assumed to be a function  
of Mach number only in this report  
(dimensionless)

$S$  = Cross-section area of the rocket vehicle, ft<sup>2</sup>

$M.N.$  = Mach number =  $V/V_{SD}$

$V_{SD}$  = Velocity of sound in the air about the rocket  
vehicle, ft/sec.

It should be noted that each stage of the rocket vehicle could have a different diameter. The diameter of the largest stage in the assembly that has not dropped off will be used.

Two drag coefficients for each configuration of the rocket vehicle are needed. It is assumed that the drag of a configuration during powered flight is different from (less than) the drag before ignition or after burnout by the amount of the base drag. The drag coefficients for

the powered and coasting conditions will be denoted by  $C_{DB}$  and  $C_{DC}$ , respectively, with numerical subscripts added to denote the configuration or stage number. Thus, up to six different drag coefficients will be needed for a three-stage rocket vehicle:  $C_{DB1}$ ,  $C_{DB2}$ ,  $C_{DB3}$ ,  $C_{DC1}$ ,  $C_{DC2}$ ,  $C_{DC3}$ .  $C_{DC1}$  and/or  $C_{DC2}$  are not needed, of course, if the first and/or second configuration do not coast.

The neglect of yaw angle in determining the drag coefficient is justified by the fact that the thrust will dominate the motion so that fairly large errors in the aerodynamics will have little effect on the trajectory. Also, the yaw angle (and its effect on the coefficients) will presumably be small.

(e) Atmosphere and wind.

The values of  $\rho$ ,  $V_{SD}$ , and  $P_a$  are available from the COESA 1962 model atmosphere<sup>15</sup> or from launch site soundings.

Actually  $V_{SD}$  is not measured directly; instead, the air temperature  $T_a$  is recorded and  $V_{SD}$  is computed from the equation

$$V_{SD} = V_{SDs} (T_a / T_{as})^{1/2} \quad (12)$$

where  $V_{SDs}$  is the standard velocity of sound (ft/sec) corresponding to standard air temperature  $T_{as}$ , and the units of  $T_a$  and  $T_{as}$  should be  $^{\circ}\text{K}$ .

If a wind is blowing, it will have an important effect on the trajectory of a multistage unguided rocket vehicle. The effect is most pronounced during the first stage, and decreases thereafter. After burnout of the last stage, the effect will be fairly small and can be neglected. The wind may be taken into account by computing the velocity components from the following equations:

$$\begin{aligned} V_X &= \dot{X} - V_{WX} \\ V_Y &= \dot{Y} - V_{WY} \\ V_Z &= \dot{Z} - V_{WZ} \end{aligned} \quad (13)$$

where  $V_{WX}$ ,  $V_{WY}$ ,  $V_{WZ}$  are the wind components in the Earth-centered axis system. The total velocity  $V$  with respect to the air mass may be computed from

$$V = (V_X^2 + V_Y^2 + V_Z^2)^{1/2} \quad (14)$$

Equations 13 follow from the definition of  $V_X$ ,  $V_Y$ , and  $V_Z$  as components of the velocity of the rocket vehicle with respect to the air mass.

It is assumed that the wind velocity is a function of  $H_G$  only and is horizontal (that is,  $V_{WZ_L} = 0$ ). If the wind velocity is exactly horizontal, then  $V_{WZ_L} \neq 0$  for  $R_{XY_L} \neq 0$ . Setting  $V_{WZ_L} = 0$  may be thought of as a flat-earth approximation, but unless  $R_{XY_L}$  is very large, it is doubtful whether  $V_W$  can be measured accurately enough for the approximation to be questionable.

The meteorological data taken before a launch should include the wind velocity  $V_W$  and the wind azimuth angle  $A_W$  as well as  $\rho$ ,  $T_a$ , and  $P_a$  vs. altitude. By convention  $A_W$  is defined to be the azimuth angle, measured clockwise from the North, from which the wind is blowing; then  $A_W$  is also the azimuth angle, measured clockwise from the South, to which the wind is blowing. It will be seen that  $V_{WX_L}$  and  $V_{WY_L}$  may be computed from the following equations:

$$\begin{aligned} V_{WX_L} &= V_W \cos (A_W - A_{X_L}) \\ V_{WY_L} &= V_W \sin (A_W - A_{X_L}) \end{aligned} \quad (15)$$

$V_{WX_L}$  and  $V_{WY_L}$  are then rotated into the Earth-centered components  $V_{WX}$ ,  $V_{WY}$  and  $V_{WZ}$ .

The presentation of the linear acceleration equations for

the burning period has now been completed.

b. Angular equations of motion.

(1) Coordinate systems.

(a) Nonrotating body axis ( $X_1, X_2, X_3$ ).

A right-hand Cartesian coordinate system which is fixed to, but does not spin with the body. The  $X_1$  axis is along the spin axis. The  $X_2$  axis lies in the vertical plane, while the  $X_3$  axis lies in the horizontal plane. This axis system is related to the reversed ( $-Y_L$ ) launch coordinate system by the two Euler angles  $\theta$  and  $\phi$ .

(b) Rotating body axis ( $X_1, X_2', X_3'$ ).

A body fixed coordinate system with the same origin as the  $X_1, X_2, X_3$  system. Axes  $X_2'$  and  $X_3'$  are along the transverse principal axis of the rocket vehicle.

(2) Development.

(a) Body axis equations.

The angular acceleration equations of the rocket vehicle may be developed from the vector equation of reference 1. The only torque considered is the overturning moment. All other moments including pitch and jet damping are assumed to be small and will be neglected; the rocket vehicle is spun to reduce the effects of any misalignments, asymmetries, or unbalances, but such a spin is not large enough to develop an appreciable Magnus moment. The vector equation of angular motion is then as follows:

$$\frac{d\vec{H}}{dt} = \frac{\partial \vec{H}}{\partial t} + \vec{\Omega} \times \vec{H} = \vec{G}_M \quad (16)$$

where

$\vec{H}$  = Angular momentum vector, lb-ft-sec

$\vec{G}_M$  = Overturning moment (vector), lb-ft

$\vec{\Omega}$  = Angular velocity of the  $X_1, X_2, X_3$  system, rad/sec



The angular velocity of the missile is denoted by  $\vec{\omega}$ ; the components of  $\vec{\omega}$  in the  $X_1, X_2, X_3$  system are  $\Omega_1 + N, \Omega_2, \Omega_3$ , where  $N$  is the axial spin of the missile and  $\Omega_1, \Omega_2, \Omega_3$  are the components of  $\vec{\Omega}$ . Then equations for  $\vec{\Omega}$  and  $\vec{H}$  are as follows: (reference 1.)

$$\begin{aligned}\vec{\Omega} &= \vec{i}_1 \Omega_1 + \vec{i}_2 \Omega_2 + \vec{i}_3 \Omega_3 \\ \vec{H} &= \vec{i}_1 A(\Omega_1 + N) + \vec{i}_2 B\Omega_2 + \vec{i}_3 B\Omega_3\end{aligned}\quad (17)$$

where

$A$  = moment of inertia of the rocket vehicle about the longitudinal principal axis, slug-ft<sup>2</sup>

$B$  = moment of inertia of the rocket vehicle about a transverse axis through the center of mass, slug-ft<sup>2</sup>

It is assumed that the mass distribution is symmetric, so the moment of inertia is the same about any transverse axis through the center of mass.

#### (b) Moments of inertia.

It is assumed that an internal-burning solid propellant is used. Approximate values of  $A$  and  $B$  during burning are computed by use of the following formulas:

$$A = A_i - k_{AP}^2 (M_i - M) \quad (18)$$

$$B = B_i - M_i (G - G_i) (G_i - G_P) - (M_i - M) k_{BP}^2 \quad (19)$$

$$\text{where } G = G_i + (G_i - G_P) (M_i - M) / M \quad (20)$$

As before, subscript  $i$  indicates values at time of ignition and  $P$  denotes a property of the propellant;  $k_{AP}$  is the radius of gyration of the propellant grain about its longitudinal axis (ft),  $k_{BP}$  is the radius of gyration of the propellant grain about a transverse axis through its center of mass (ft), and  $G$  is the distance from the base to the center of mass of the rocket vehicle (ft). The quantities  $G_P, k_{AP}$  and  $k_{BP}$  are assumed to be constant for an internal burning grain;  $A_i, B_i, G_i$ , and  $M_i$  are, of course, constant, so  $M$  is the only variable.

Equations 19 and 20 cannot be used for an end-burning grain unless formulas are added for  $k_{BP}$  and  $G_P$ . None of these equations apply for liquid propellant rockets.

(c) Aerodynamic moment.

The expression for  $\vec{G}_M$  is as follows:

$$\vec{G}_M = \frac{1}{2} \rho d S V C_{Ma} (\vec{i}_2 V_3 - \vec{i}_3 V_2) \quad (21)$$

For the present application  $C_{Ma}$  is the static stability derivative (dimensionless) and is a negative number which can be computed by use of the equation:

$$C_{Ma} = \frac{C_{Na} (C_p - G)}{d} \quad (22)$$

where

$C_{Na}$  = Normal force coefficient (dimensionless)

$C_p$  = Distance from the base of the rocket to the normal force center of pressure, feet

In this report  $C_{Ma}$ ,  $C_{Na}$ , and  $C_p$ , are assumed to be functions of Mach number only. In the trajectory-computation program these functions and  $C_D$  are evaluated by table look-up and interpolation.

(d) Euler angle relation.

If equations 17 and 21 are substituted into equation 16, the  $X_2$  and  $X_3$  components of the resulting vector equation will be as follows:

$$\begin{aligned} B\dot{\Omega}_2 + \dot{B}\Omega_2 + (A - B)\Omega_1\Omega_3 + AN\Omega_3 &= \frac{1}{2}\rho VdS C_{Ma} V_3 \\ B\dot{\Omega}_3 + \dot{B}\Omega_3 + (B - A)\Omega_1\Omega_2 - AN\Omega_2 &= -\frac{1}{2}\rho VdS C_{Ma} V_2 \end{aligned} \quad (23)$$

It is desirable to replace  $\Omega_1$ ,  $\Omega_2$ ,  $\Omega_3$ ,  $\dot{\Omega}_2$ ,  $\dot{\Omega}_3$  in these equations by functions of  $\phi$  and  $\theta$ . This may be done by use of the following relations:

$$\Omega_1 = -\dot{\phi} \sin \theta$$

$$\begin{aligned}\Omega_2 &= -\dot{\theta} \\ \Omega_3 &= -\dot{\phi} \cos \theta\end{aligned}$$

(24)

These equations neglect the angular velocity of the Earth ( $\omega_E$ ); they are written by inspection of figure 2. It will be seen that

$$\begin{aligned}\dot{\Omega}_2 &= -\ddot{\theta} \\ \dot{\Omega}_3 &= -\ddot{\phi} \cos \theta + \dot{\phi} \dot{\theta} \sin \theta\end{aligned}$$

(25)

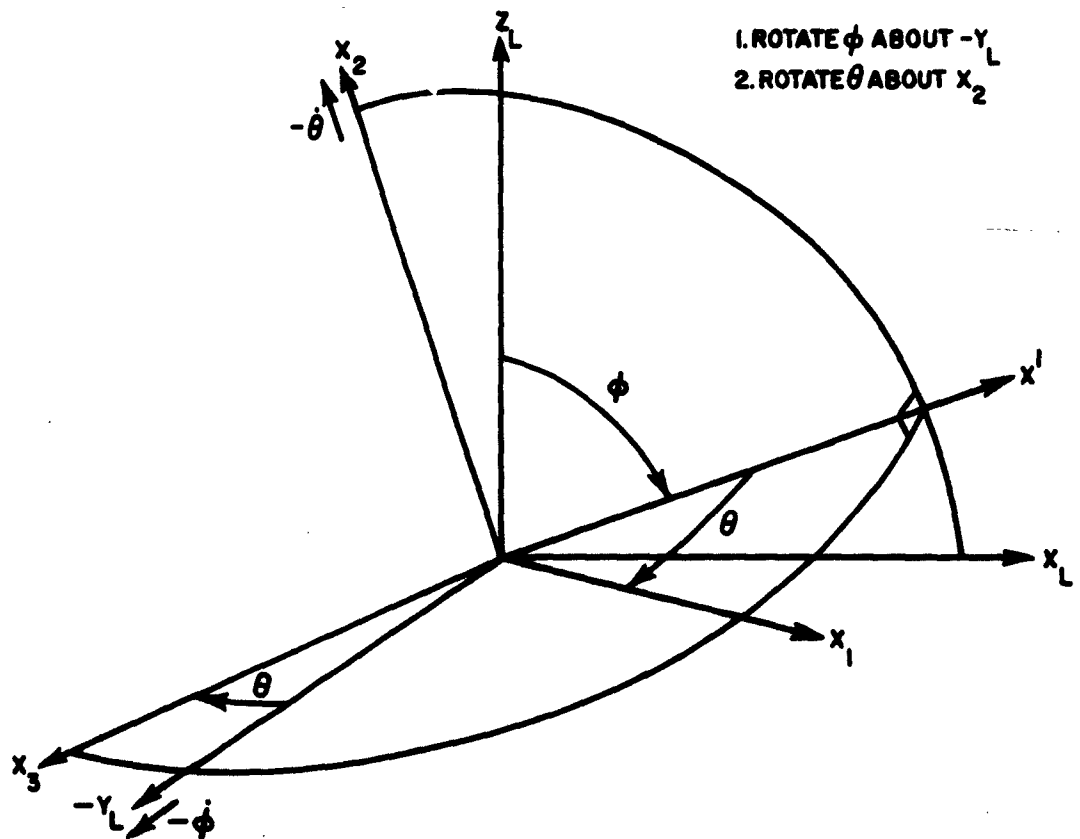


Figure 2. Coordinate system for angular equations.

Substitution of equations 24 and 25 into equations 23 yields the required angular acceleration equations as follows:

$$\begin{aligned}
 & -B\ddot{\theta} - \dot{B}\dot{\theta} + (A - B)\dot{\varphi}^2 \sin \theta \cos \theta - AN\dot{\varphi} \cos \theta \\
 & = \frac{1}{2}\rho V S d C_{M\alpha} V_3 \\
 & -B\ddot{\varphi} \cos \theta - \dot{B}\dot{\varphi} \cos \theta + (2B - A)\dot{\varphi}\dot{\theta} \sin \theta + AN\dot{\theta} \\
 & = -\frac{1}{2}\rho V S d C_{M\alpha} V_2
 \end{aligned} \tag{26}$$

These equations are dynamically exact, except for the neglect of  $\omega_E$  which is negligibly small compared to  $\dot{\varphi}$ ,  $\dot{\theta}$ , and  $N$ .

Equations 26 may be put in a form more suitable for computation by dividing through by  $B$ . The equations become

$$\begin{aligned}
 & \ddot{\theta} + (\dot{B}/B)\dot{\theta} + (1 - (A/B))\dot{\varphi}^2 \sin \theta \cos \theta + (A/B)N\dot{\varphi} \cos \theta \\
 & = (-\frac{1}{2}\rho V d S C_{M\alpha}) \frac{V V_3}{B} \\
 & \ddot{\varphi} \cos \theta + (\dot{B}/B)\dot{\varphi} \cos \theta - (2 - (A/B))\dot{\varphi}\dot{\theta} \sin \theta - (A/B)N\dot{\theta} \\
 & = (\frac{1}{2}\rho d S C_{M\alpha}) \frac{V V_2}{B}
 \end{aligned} \tag{27}$$

(e)  $\dot{B}$  Terms.

For an internal burning solid-propellant rocket, the following formulas are used to compute the  $\dot{B}$  terms:

$$\dot{B} = \dot{M} \left[ (M_i^2/M^2) (G_i - G_P)^2 + k_{BP}^2 \right]$$

where

$$\dot{M} = -T_{Tv} \left\{ M_{Pl} / \int_{t_l}^{t_b} T_{Tv} dt \right\} \tag{28}$$

These formulas are easily derived from equations 9 and 19.

## (f) Body-cross wind components.

A coordinate transformation is needed to compute  $V_2$  and  $V_3$  from  $V_{X_L}$ ,  $V_{Y_L}$ ,  $V_{Z_L}$ . The following equations can be written from inspection of figure 2:

$$V_2 = -V_{X_L} \cos \varphi + V_{Z_L} \sin \varphi$$

$$V_3 = -V_{X_L} \sin \varphi \sin \theta - V_{Y_L} \cos \theta - V_{Z_L} \cos \varphi \sin \theta \quad (29)$$

This completes the derivation of the equations of motion during burning.

c. Summary of equations of motion.

If the rocket is assumed to have thrust misalignments, additional terms are added to the equations to account for the new forces and moments created. For convenience, the equations are restated here with the new terms added.

$$\ddot{X} = \frac{1}{M} \left[ T_x - \frac{D\dot{X}}{R} \right] + G_x + 2\omega_E \dot{Y} + \omega_E^2 X$$

$$\ddot{Y} = \frac{1}{M} \left[ T_y - \frac{D\dot{Y}}{R} \right] + G_y - 2\omega_E \dot{X} + \omega_E^2 Y$$

$$\ddot{Z} = \frac{1}{M} \left[ T_z - \frac{D\dot{Z}}{R} \right] + G_z$$

$$\ddot{\varphi} = -\frac{\dot{B}}{B} \dot{\varphi} + \left\{ \left[ \left( 2 - \frac{A}{B} \right) \dot{\varphi} \sin \theta + \frac{A}{B} N \right] \dot{\theta} + \frac{1}{8} \rho C_{Na} (C_P - G) \frac{D^2 V V_2}{B} + \frac{T G \delta_T \cos \varphi A}{B} \right\} \frac{1}{\cos \theta}$$

$$\ddot{\theta} = -\frac{\dot{B}}{B} \dot{\theta} - \left[ \left( 1 - \frac{A}{B} \right) \dot{\varphi} \sin \theta + \frac{A}{B} N \right] \dot{\varphi} \cos \theta - \frac{1}{8} \rho C_{Na} (C_P - G) \frac{D^2 V V_3}{B} - \frac{T G \delta_T \sin \varphi A}{B} \quad (30)$$

where

$\delta_T$  = thrust misalignment angle, radians

$\varphi_T$  = orientation angle of jet misalignment force; measured in the  $X'_2 - X'_3$  plane from the  $X'_2$  axis and positive in the sense of a counter clockwise rotation as seen from the positive  $X'_1$  axis, radians.

$\varphi_A = \theta_T + \int_{t_i}^t N dt$  measured in the  $X_2 - X_3$  plane from the  $X_2$  axis and positive in the sense of a counterclockwise rotation as seen from the positive  $X_1$  axis, radians.

These equations require a table of roll rate ( $N = \dot{\phi}$ ) vs. time.

The thrust vector is given in the launch coordinate system and rotated to the Earth-centered system by the use of a matrix rotation. The thrust vector in the launch coordinate system is

$$\begin{aligned} T_{xL} &= T \left[ \cos \theta \sin \varphi - \delta_T (\cos \varphi_A \cos \varphi + \sin \varphi_A \sin \varphi \sin \theta) \right] \\ T_{yL} &= -T \left[ \sin \theta + \delta_T \sin \varphi_A \cos \theta \right] \\ T_{zL} &= T \left[ \cos \theta \cos \varphi + \delta_T (\cos \varphi_A \sin \varphi - \sin \varphi_A \cos \varphi \sin \theta) \right] \end{aligned} \quad (31)$$

d. Geocentric relationships.

(1) Shape of the Earth.

Figure 3 shows a meridian section of the earth where  $a_E$  is the semi-major (equatorial) axis,  $\theta_c$  is the geocentric latitude, and  $R_E$  is a radius vector from center to the surface of the earth.  $R_E$  is a function of the geocentric latitude and is given as

$$R_E = \frac{a_E}{(1 + P_E \sin^2 \theta_c)^{1/2}}$$

where

$$P_E = \frac{e_E^2}{1 - e_E^2}$$

(32)

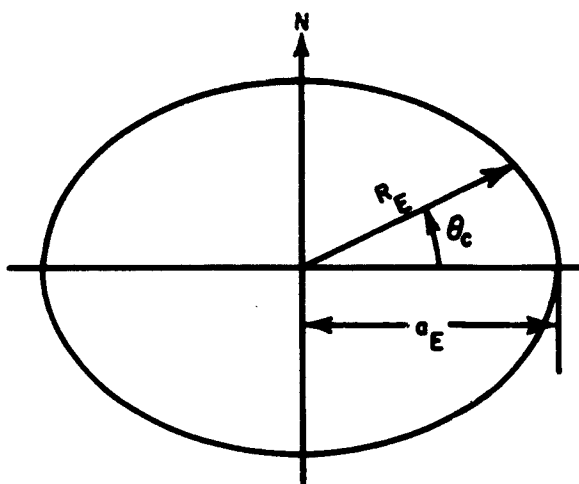
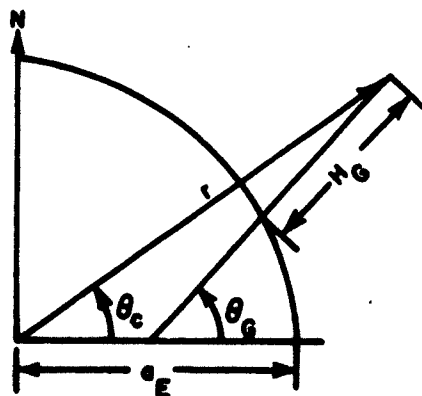


Figure 3

(2) Geodetic sublatitude and altitude.

Figure 4 shows the geometric relation between the geodetic and geocentric latitude and altitude.



$\theta_G$  - Geodetic  
Latitude

$H_G$  - Geodetic  
Altitude

Figure 4

To convert geodetic latitude and altitude to geocentric latitude and radius:  
(reference 6)

$$\tan \theta_c = \left[ \frac{S + H_G}{C + H_G} \right] \tan \theta_G \quad (33)$$

where

$$C \triangleq \frac{a_E}{(1 - e_E^2 \sin^2 \theta_G)^{1/2}} \quad S \triangleq C(1 - e_E^2) \quad (34)$$

$$R = \left[ (C + H_G)^2 \cos^2 \theta_G + (S + H_G)^2 \sin^2 \theta_G \right]^{1/2} \quad (35)$$

to convert geocentric latitude and radius to geodetic latitude and altitude:  
(reference 7)

$$\begin{aligned} \theta_G &= \theta_c + \sin^{-1} \left\{ \frac{a_E}{R} \left[ f \sin 2 \theta_c + f^2 \sin 4 \theta_c \left( \frac{a_E}{R} - \frac{1}{4} \right) \right] \right\} \\ H_G &= R - a_E \left[ 1 - f \sin^2 \theta_c - \frac{f^2}{2} \sin^2 2 \theta_c \left( \frac{a_E}{R} - \frac{1}{4} \right) \right] \end{aligned} \quad (36)$$

The following geocentric constants were used in the program.

Adopted Geocentric Constants (1961) (reference 8)

$$\begin{aligned} a_E &= 20,925,647.12 \text{ ft} \\ GM &= 1.4076427 \times 10^{16} \text{ ft}^3/\text{sec}^2 \\ &= 2.316686 \times 10^{12} \text{ NM} \left( \frac{\text{ft}}{\text{sec}} \right)^2 \\ g_E &= 32.174 \text{ ft/sec}^2 \\ 1/f &= 298.30 \pm 0.05 \end{aligned}$$



TDR-63-11

$$\epsilon_E = 0.0818133302$$

$$J = (1623.42 \pm 0.5) \times 10^{-6}$$

$$P_E = 0.00673852$$

$$\omega_E = 7.292115 \times 10^{-5} \text{ rad/sec}$$

3. DATA INPUT AND PROGRAM USAGE.

INPUT DATA

All input data are read into SPURT at one time and in one "read" block. This includes data for the subroutines (two-body and impact). Most data, except for special cases and control integers, are read in a FORTRAN 7F10.0 format. This format is for seven data words (of ten digits each) per card. The decimal point is always punched in the field. On Repeat Blocks, as many words will be read as specified by a previously read integer.

TDR-63-11

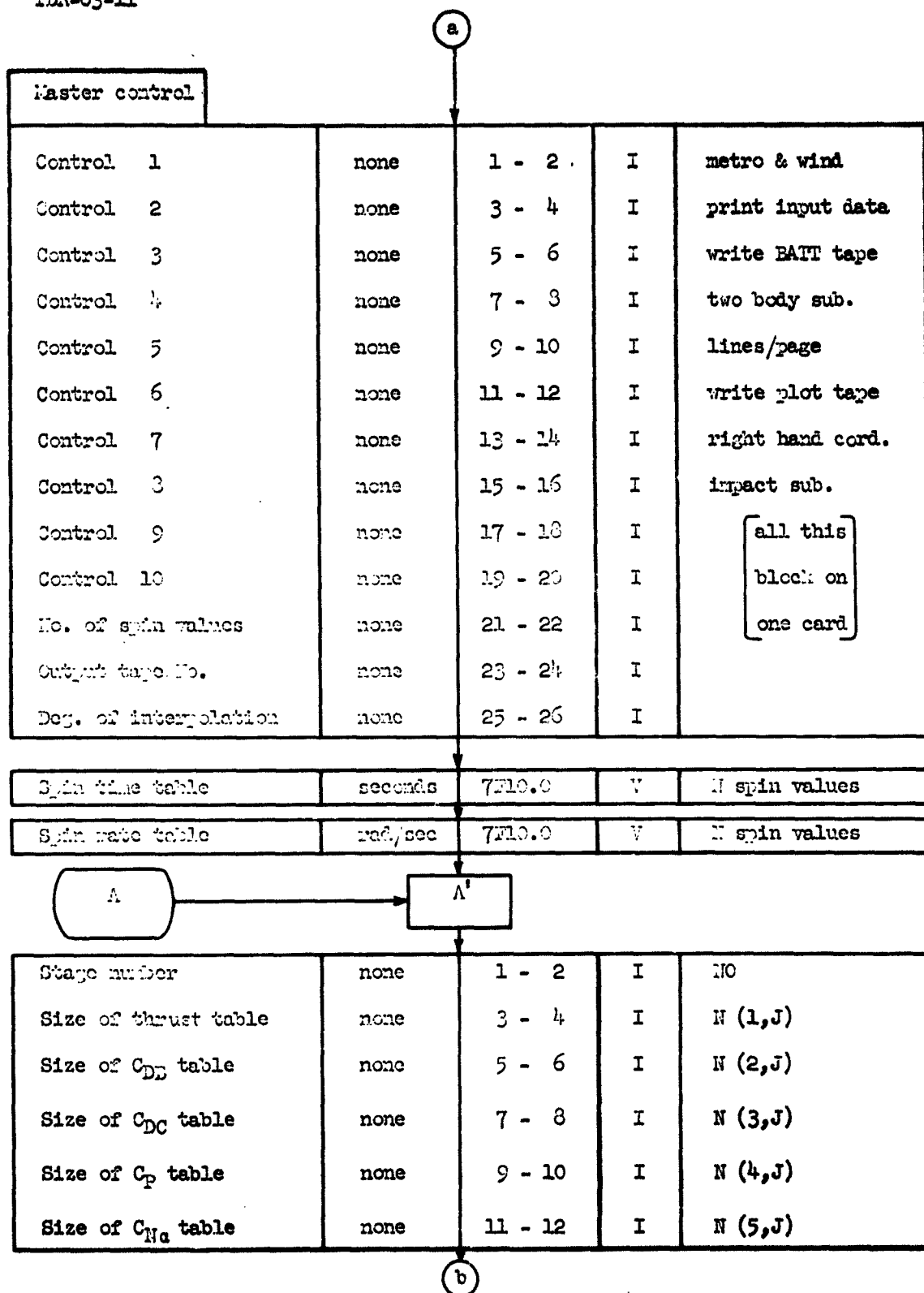
FLOW CHART OF INPUT DATA:

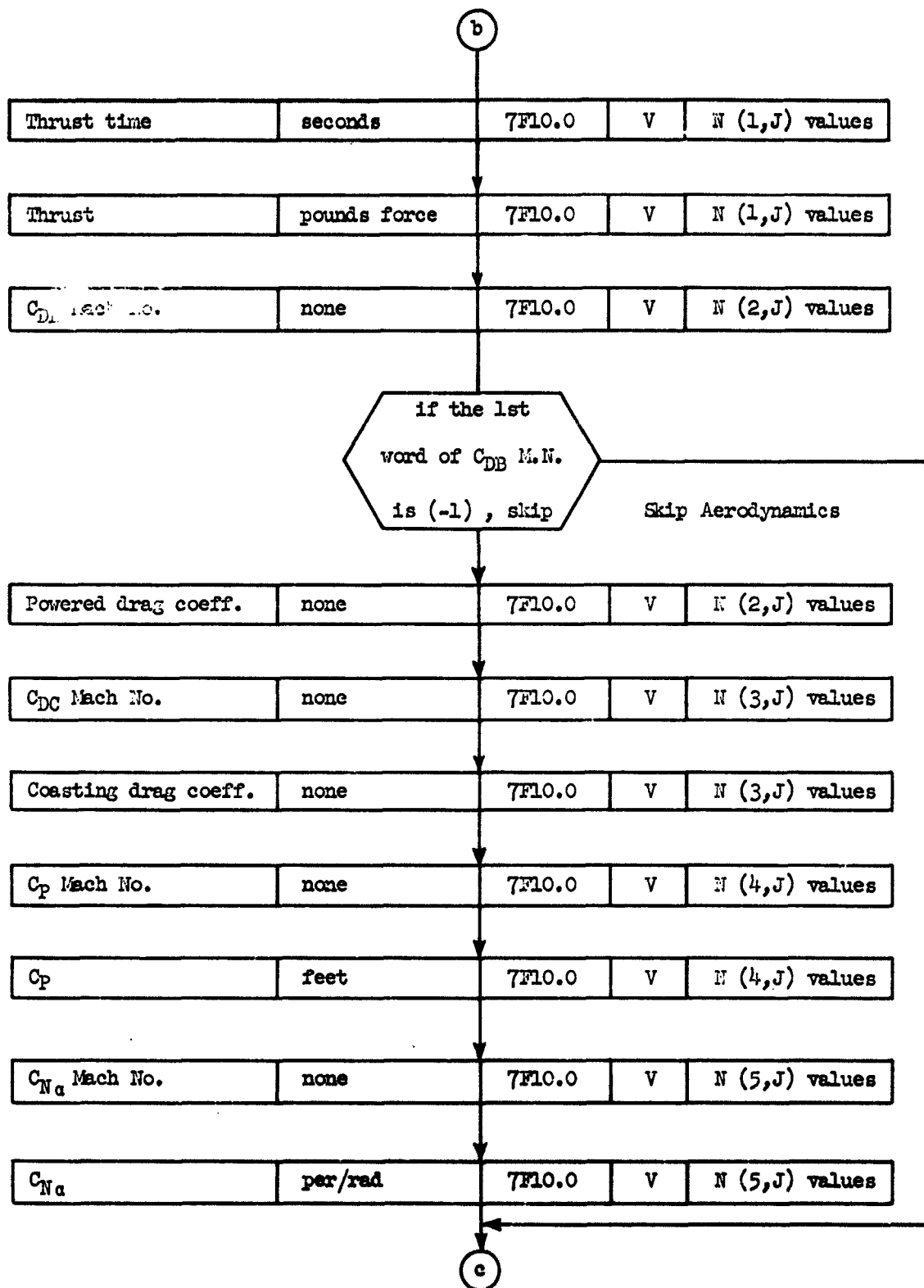
Parameter	Dimension	Column	Mode	Remarks
No. of stages	none	1 - 2	I	1 Card
Payload weight	pounds	3 - 15	V	

Name	none	1 - 80	Hollerith	1 Card
------	------	--------	-----------	--------

Initial latitude	degrees (+ N)	1 - 10	V	1 Card
Initial longitude	degrees (+ E)	11 - 20	V	
Initial altitude	feet	21 - 30	V	
Initial azimuth	degrees CW from N	31 - 40	V	
Initial X	feet	41 - 50	V	
Initial Y	feet	51 - 60	V	1 Card
Initial Z	feet	61 - 70	V	
Initial $\dot{X}$	feet/sec	1 - 10	V	
Initial $\dot{Y}$	feet/sec	11 - 20	V	
Initial $\dot{Z}$	feet/sec	21 - 30	V	
Initial $\phi$	degrees	31 - 40	V	1 Card
Initial $\theta$	degrees	41 - 50	V	
Initial $\dot{\phi}$	deg/sec	51 - 60	V	
Initial $\dot{\theta}$	deg/sec	61 - 70	V	
Start time = 0.	seconds	1 - 10	V	
Time of last stage B.O.	seconds	11 - 20	V	1 Card

2



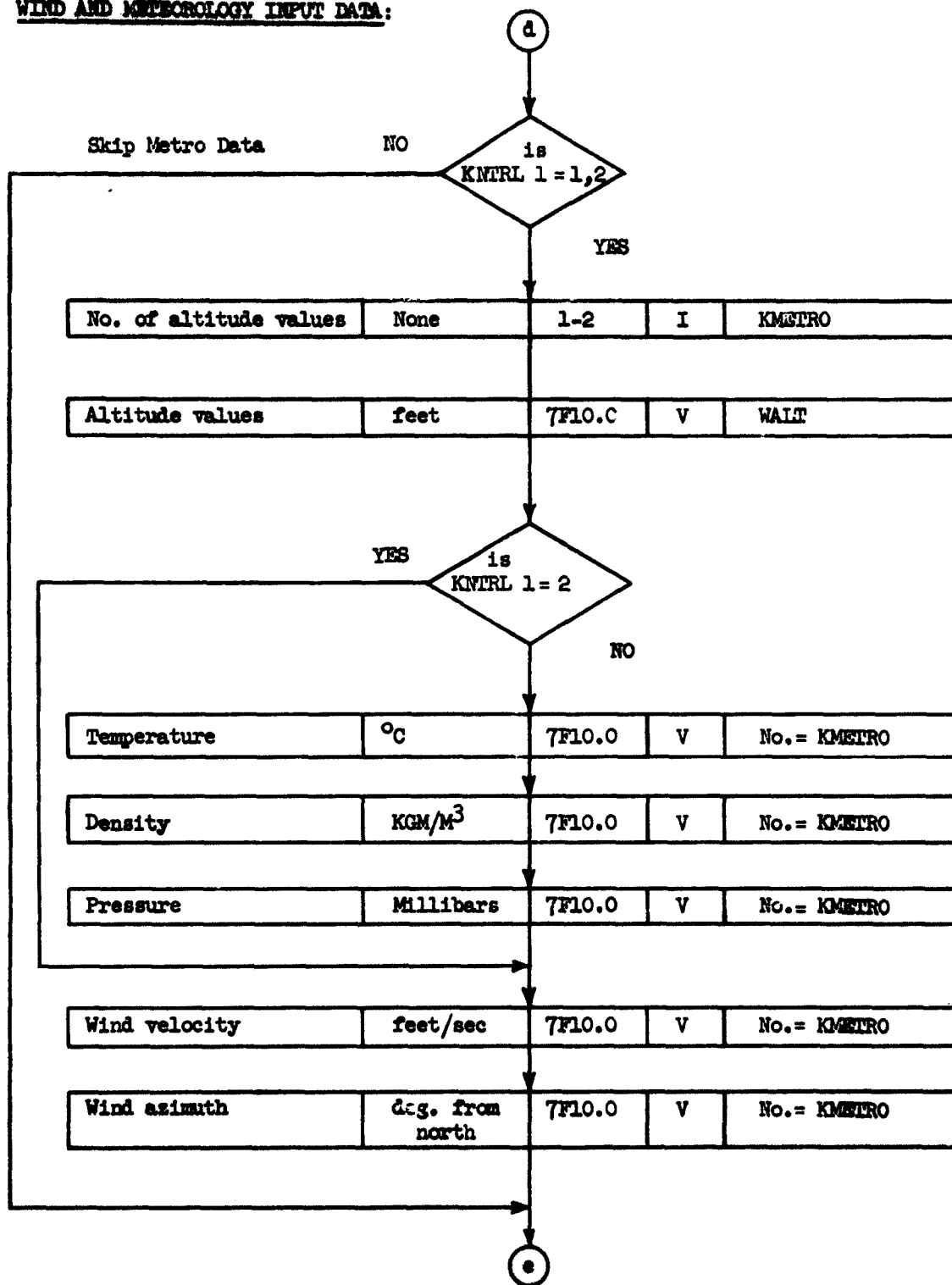


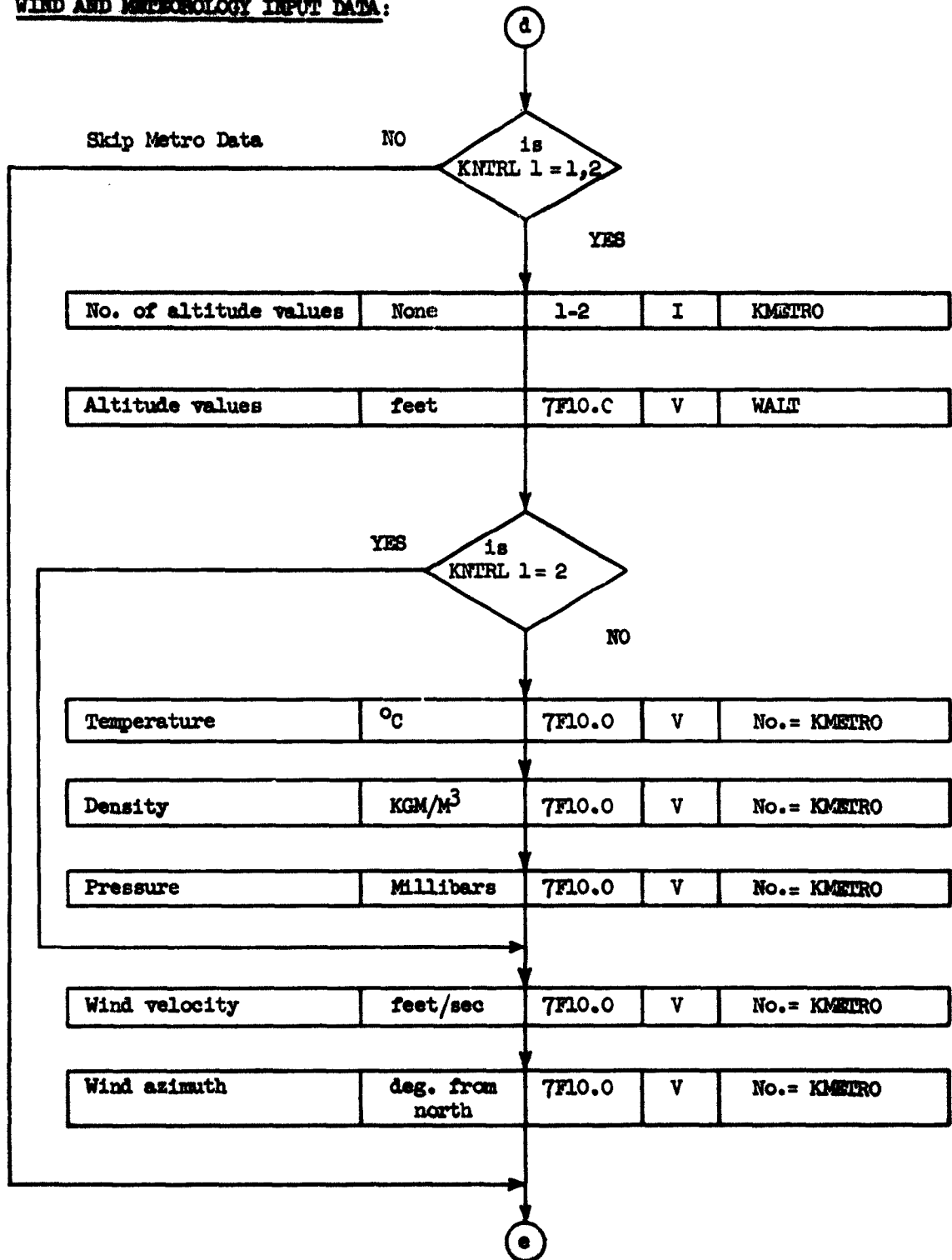
c

Pressure at thrust meas.	pounds/in <sup>2</sup>	1 - 10	V	1 Card
Exit area	inch <sup>2</sup>	11 - 20	V	
Stage diameter	feet	21 - 30	V	
Missile C.G.	feet	31 - 40	V	
Stage fuel C.G.	feet	41 - 50	V	
Fuel axial I/M	feet <sup>2</sup>	51 - 60	V	
Fuel transverse I/M	feet <sup>2</sup>	61 - 70	V	
Missile axial I	slug-ft <sup>2</sup>	1 - 10	V	
Missile transverse I	slug-ft <sup>2</sup>	11 - 20	V	
Stage weight	pounds	21 - 30	V	
Stage consumed wt	pounds	31 - 40	V	1 Card
Orientation of T.M.A.	radians	41 - 50	V	
Thrust misalign angle	radians	51 - 60	V	1 Card
- OMIT -		61 - 70	V	
Ignition time from launch	seconds	1 - 10	V	
Burnout time from launch	seconds	11 - 20	V	1 Card
TMCC from launch	seconds	21 - 31	V	



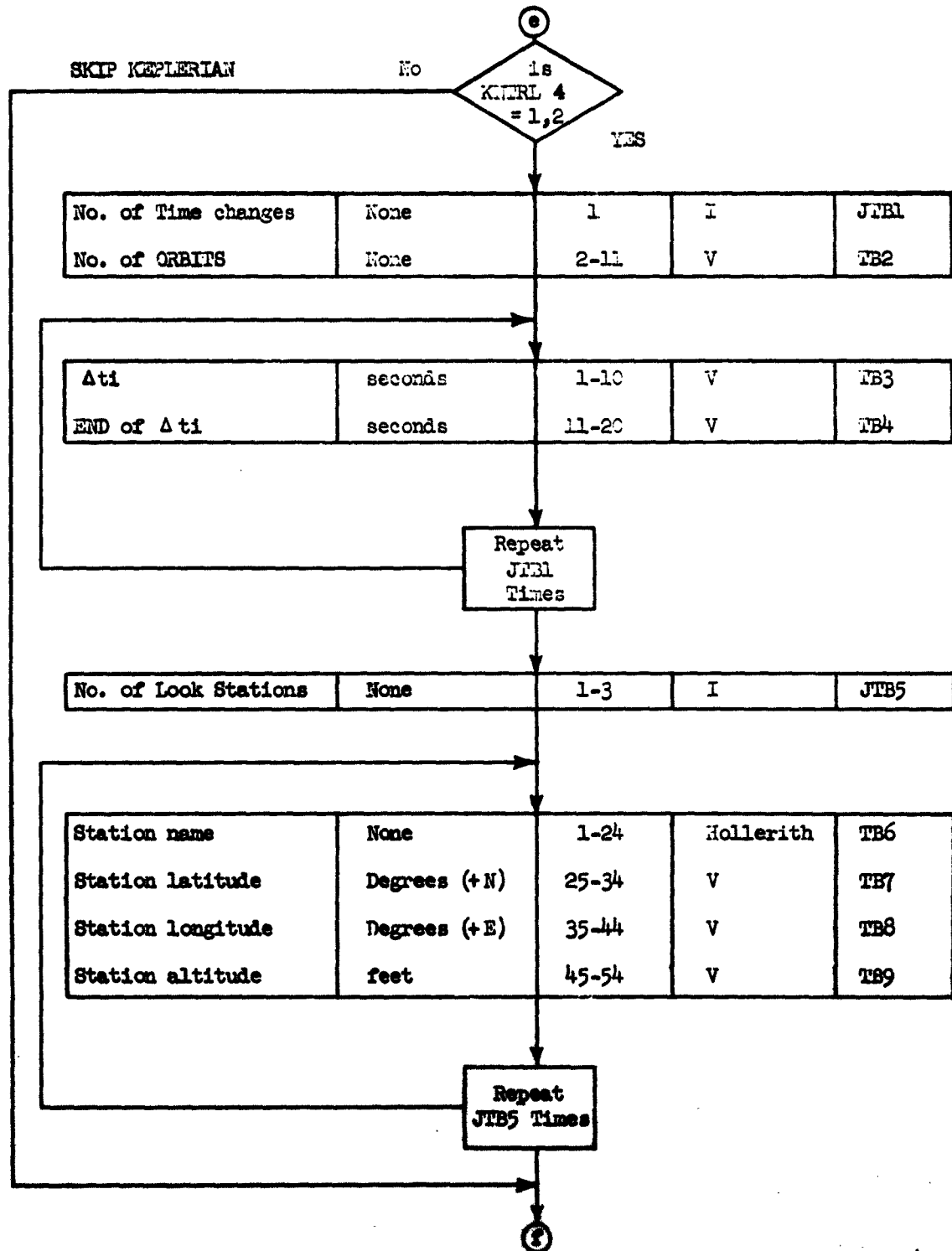
d

WIND AND METEOROLOGY INPUT DATA:

WIND AND METEOROLOGY INPUT DATA:

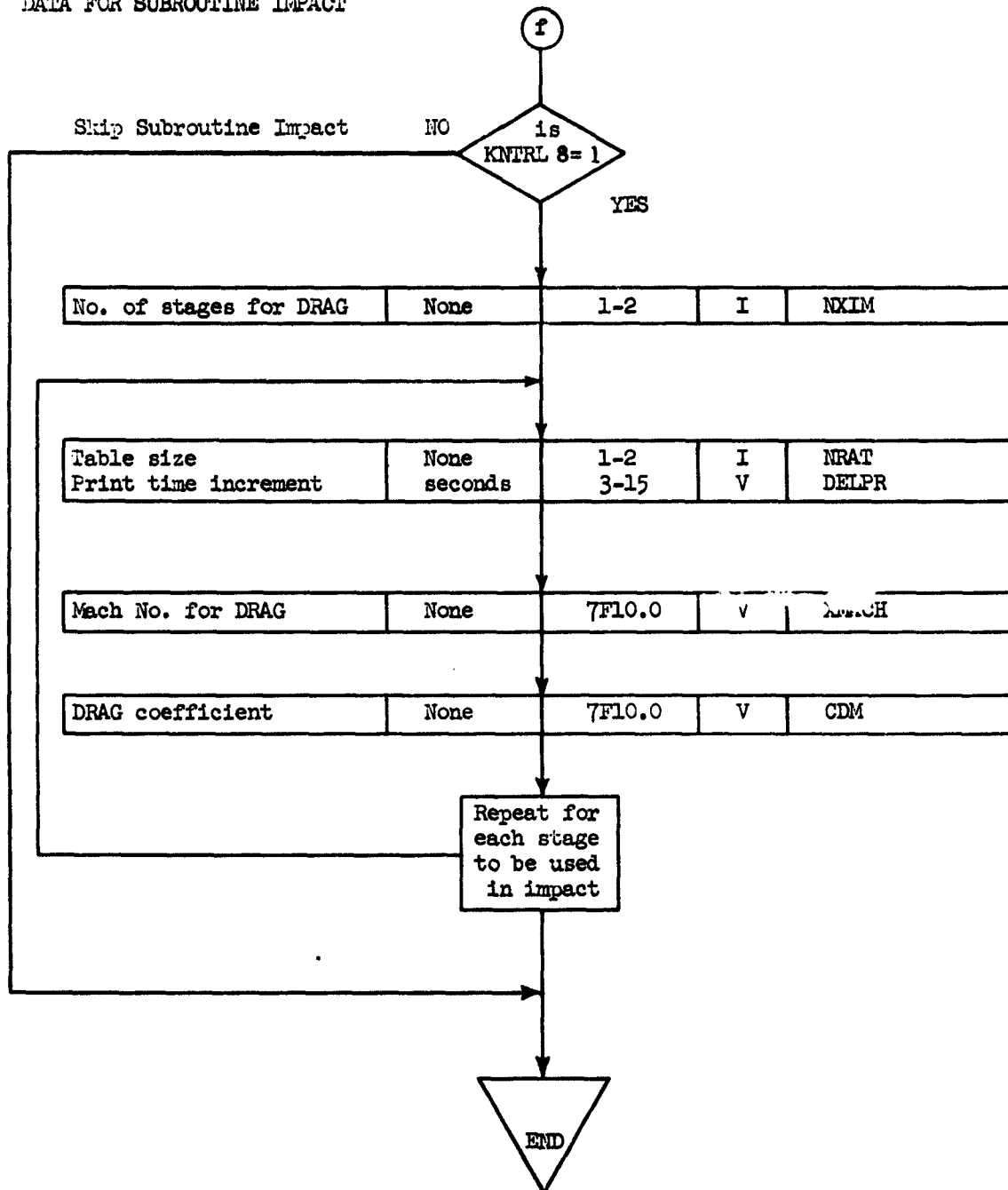


① INPUT DATA FOR KEPLERIAN TRAJECTORY AND LOOK ANGLES:



TDR-63-11

DATA FOR SUBROUTINE IMPACT



I ~ INTEGER - Fixed point numbers  
V ~ VARIABLE - Floating point numbers

### MASTER CONTROL NUMBERS

These numbers control various options of the program as follows:

Control 1. When KNTRL(1) = 1\*, the program will read in the temperature, pressure, and density and use them to compute the aerodynamic forces and moments. Otherwise, the standard 1962 atmosphere is used.

When KNTRL(1) = 1 or 2, the program will read in wind velocity and wind azimuth vs. altitude. The program will compute the path of the missile due to the winds.

Control 2. When KNTRL(2) = 1, the input data are printed out.

Control 3. When KNTRL(3) = N, the program will write tape number N for input to the BATT program.

Control 4. When KNTRL(4) = 1, the program will use the Keplerian (TWO-BOD) subroutine for coasting flight on the last stage. If KNTRL(4) = 2, it will use it on all stages.

Control 5. When KNTRL(5) = NO, the program will write NO lines of output per page.

Control 6. When KNTRL(6) = N, the program will prepare an input tape (N) for the plot program.

Control 7. When KNTRL(7) = 1, the launch coordinate system printed out is a right-hand one instead of a left-hand one.

Control 8. If KNTRL(8) = 1, the integrating unpowered flight trajectory is computed for all stages except the last one. If KNTRL(8) = 2, the trajectory is computed for all stages.

Control 9. Not used.

Control 10. Not used.

---

\* When a KNTRL number is left blank, that option will be ignored.

4. VARIABLES USED IN SPURT.INPUTS

A	*	Initial axial moment of inertia for each stage (slug-ft <sup>2</sup> )
AALT	*	Initial geodetic altitude (ft)
AAZIM	*	Initial launch azimuth (deg)
AAZI2		Initial launch azimuth (rad)
ACODE		Integration code
ACODES		Integration code table
AE	*	Exit area of rocket for each stage (in <sup>2</sup> )
ALAT	*	Initial geodetic latitude (deg)
ALA2		Initial geodetic latitude (rad)
ALON	*	Initial longitude (deg)
ALO2		Initial longitude (rad) +
ALT		Geocentric altitude (ft)
AN1		$\frac{\lambda}{2}$ + longitude (rad)
AN2		Colatitude (rad)
AXLM		Axial moment of inertia (slug-ft <sup>2</sup> )
AXLMB		Axial moment of inertia over transverse moment of inertia
B	*	Initial transverse moment of inertia for each stage (slug-ft <sup>2</sup> )
BAZIM		$AAZI2 - \frac{\lambda}{2}$ (rad)
BDOT		Rate of change of the transverse moment of inertia, B (slug-ft <sup>2</sup> /sec)
BDOTB		$\dot{B} / B$ (sec <sup>-1</sup> )
BL		Integration data storage
BLON		$\frac{\lambda}{2} + ALO2$ (rad)
C		$\frac{A}{E} REE / \sqrt{1 - e_E^2 \sin^2 \theta_G}$ (ft)
CALAT		Cosine of the initial latitude
CBLOCK	*	Common block for integration
CDM	*	$C_D A / m$ - drag parameter used for empty stages (ft <sup>2</sup> /slugs) +

+ Stored in common

INPUTS

CHECK		Flag for integration check
CN	*	Input table of normal force coefficient $C_{N_a}$ ( $\text{rad}^{-1}$ )
CNI		Normal force coefficient used in calculation
CNMACH	*	Input CN Mach table
CNMTAB		Inverted CN Mach table
CNTAB		Inverted CN table
COEF		Coefficient used in aerodynamic moment = $C_{N_a} (C_p - C_G) \frac{D^2 V_\infty}{B}$
COLAT		Initial co-latitude = $\frac{\pi}{2} - \text{ALA2}$ (rad)
COTHC		Co-geocentric latitude = $\frac{\pi}{2} - \text{THC}$ (rad)
CP	*	Initial center of pressure table (feet from tail)
CPHI		Cosine of PHI
CPHT		Cosine of PHT
CPI		Center of pressure used in calculation
CPMACH	*	Initial Mach No. table for center of pressure
CPMTAB		Inverted CP Mach No. table
CPTAB		Inverted CP table (ft)
CTHC		Cosine of initial geocentric latitude = $\text{Cos}(\text{THC}) +$
CTHET		Cosine of THETA
D	*	Diameter of each stage (ft)
D1MACH	*	Input Mach No. table for burning drag coefficient
D2MACH	*	Input Mach No. table for coasting drag coefficient
DC		Direction cosines matrix
DELPR	*	Print times used in impact (sec) +
DELTH		Difference between $\theta_c$ and $\theta_g$ (rad)
DMTAB		Inverted drag Mach table
DRAG1	*	Input table of burning drag coefficient
DRAG2	*	Input table of coasting drag coefficient
DT		Print interval (sec)
DTAB		Inverted drag table

+ Stored in common

INPUTS

DUM	*	Dummy variable	
ERR		Integration error	
FDRAG		Drag force (lb)	
FORCE		Vacuum thrust (lb)	
FT		Thrust on vehicle (lb)	
FTT		Total impulse (lb - sec)	
GI		Center of gravity at any given time (feet from tail)	
GK		Earth oblateness term $a_E^2 J$ (ft <sup>2</sup> )	
GM		Gravitational constant for the Earth (ft <sup>3</sup> /sec <sup>2</sup> )	
GO	*	Initial CG of the remaining missile for a given state (ft)	
GOP		Difference between GO and GP (ft)	
GP	*	Initial CG of the fuel for a given stage (ft)	
GRAVO		Gravity constant (32.174 ft/sec <sup>2</sup> )	
GRV		Term used to compute gravity components	
GRVX		Gravity component along the X axis (ft/sec <sup>2</sup> )	
GRVY		Gravity component along the Y axis (ft/sec <sup>2</sup> )	
GRVZ		Gravity component along the Z axis (ft/sec <sup>2</sup> )	
H		Velocity vector (no wind) (ft/sec)	
HALFPI		$\frac{\pi}{2}$	
I		Utility index	
ICODE		Code used to determine integration method	
II		Utility index	
III		Utility index	
INTN	*	Order of interpolation	
IRA		Utility index	
IT		Print table count	
J		Utility index	
JP		Lines per page count for output	
JTB1	*	Two Body input (number of time changes)	+
JTB5	*	Two Body input (number of look-angle stations)	+
K		Utility index	
KBATT		Tape number for BATT tape = KNTRL(3)	+
KIX		Number of stages for subroutine impact	

+ Stored in common

INPUTS

KMETRO	*	Size of Metro tables	
KNTRL(10)	*	Controls	
KPLOT		Tape number of PLOT tape = KNTRL(6)	
L		Stage count	
LSKIP		Skip aerodynamics	
N	*	Table size	
NO	*	Input card check	
N1		Thrust table size for different stages	
N2		Burning drag table size for different stages	
N3		Coasting drag table size for different stages	
N4		CP table size for different stages	
N5		CN table size for different stages	
NAME	*	Page title (up to 80 Hollerith characters)	+
NOE		Number of equations used in integration	
NOT	*	Output tape number	+
NNX	*	Table of NRAT values	+
NRAT	*	Table size for each stage in impact	
NS	*	Number of stages ( $NS \leq 10$ )	
NSPIN	*	Spin table size ( $NSPIN \leq 100$ )	
NXIM	*	Number of stages read in impact drag table	
OMEG		Spin rate of Earth = $\omega_E$ (rad/sec)	
OMEG2		Earth spin rate squared = $\omega_E^2$ (rad <sup>2</sup> /sec <sup>2</sup> )	
OUT		Variable used for output	
P1		Pressure times exit area (lb)	
PAT	*	Pressure at which thrust is measured (lb/in <sup>2</sup> )	
PHI		Euler angle used in equations (rad)	
PHID		First derivative of PHI (rad/sec)	
PHIDD		Second derivative of PHI (rad/sec <sup>2</sup> )	
PHI	*	Orientation angle of thrust misalignment (rad)	
PI		$\pi$	
PRES		Atmospheric pressure (lb/ft <sup>2</sup> )	
PRTIME		Time at which to print (sec)	

+ Stored in common

INPUTS

PWGT	*	Stage input weight (lb)
PWGTC	*	Stage fuel weight (lb)
PX		Earth centered position vector used in integration (ft)
PXD		First derivative of position vector (ft/sec)
PXDD		Second derivative of position vector (ft/sec <sup>2</sup> )
PXL		Position vector in launch coordinate system (ft)
PXLD		Velocity vector in launch coordinate system (ft/sec)
PXLDD		Acceleration vector in launch coordinate system (ft/sec <sup>2</sup> )
PXND		Velocity vector in local coordinate system (ft/sec)
PYL WGT	*	Payload weight (lb)
R		Distance from Earth center to vehicle (ft)
RA	*	Fuel axial radius of gyration squared for each stage (ft <sup>2</sup> )
RANGE		Range at burnout of each stage (N.M.) +
RANGE1		Dummy variable used for impact (N.M.)
RAD		$\frac{\pi}{180}$
RB	*	Fuel transverse radius of gyration squared for each stage (ft <sup>2</sup> )
RE		Radius of Earth as a function of latitude (ft)
REE		Equatorial radius of the Earth (ft)
REL		Distance from Earth center to launch pt (ft) +
RHO		Atmospheric density (slugs/ft <sup>3</sup> )
ROT		Matrix from launch to Earth centered coordinates
ROT1		"COMMON" rotation matrix +
R1X		Dummy variable used in X integration
R2X		Dummy variable used in Y integration
R3X		Dummy variable used in Z integration
R4X		Dummy variable used in $\phi$ integration
R5X		Dummy variable used in $\theta$ integration
R2		Distance squared from Earth center to vehicle (ft <sup>2</sup> )
S		$\frac{A}{C} (1 - e_E^2) (ft)$
SILAT		Sine of the initial latitude
+ Stored in common		



INPUTS

SMAX		Maximum integration step size (sec)	
SMIN		Minimum integration step size (sec)	
SPHI		Sine of PHI	
SPHT		Sine of PHT	
SPI		Inverted spin table	
SPIN	*	Input spin table (rad/sec)	
SPIT		Integrated spin table (rad)	
SPT		Inverted spin time table	
SPTIME	*	Input spin time table (sec)	
SS		Integration step size (sec)	
STARTT	*	Start time or launch time (sec)	
STHC		Sine of the initial geocentric latitude	+
STHET		Sine of THETA	
STP	*	Initial PHI (deg)	
STPD	*	Initial PHI DOT (deg/sec)	
STT	*	Initial THETA (deg)	
STTD	*	Initial THETA DOT (deg/sec)	
STX	*	Initial position vector in launch coordinate system (ft)	
STXD	*	Initial velocity vector in launch coordinate system (ft/sec)	
SW		Angle between wind azimuth and X axis (rad)	
T		Thrust vector in launch coordinate system (lb)	
TABLE		Table of printout times	
TB2	*	Number of orbits	+
TB3	*	Time increment $\Delta t$ (sec)	+
TB4	*	Ending time (sec)	+
TB6	*	Name of look-angle station (up to 24 Hollerith characters)	+
TB7	*	Latitude of look-angle station (deg)	+
TB8	*	Longitude of look-angle station (deg)	+
TB9	*	Altitude of look-angle station (ft)	+
TDEL	*	Thrust misalignment angle (rad)	
TFMP		Stage drop time (sec)	+
THC		Initial geocentric latitude (rad)	

+ Stored in common

1

INPUTS

THETA		Euler angle in the horizontal plane (rad)
THETD		First derivative of THETA (rad/sec)
THETDD		Second derivative of THETA (rad/sec <sup>2</sup> )
THRUST	*	Input thrust table (lb)
THTAB		Inverted thrust table
TIME		Time (dependent variable) (sec)
TIMET	*	Time table for thrust (sec)
TMBO	*	Burnout time for each stage (sec)
TMCC	*	Time to change coefficients for each stage (sec)
TMI	*	Ignition time for each stage (sec)
TOMEG		Twice the Earth spin rate = $2\omega_E$ (rad/sec)
TP		Intermediate variables used in output block
TP1		Intermediate variables used in output block
TP2		Intermediate variables used in output block
TPHO		Orientation angle of thrust misalignment at a given time (rad)
TRERR		Integration truncation error
TRVM		Transverse moment of inertia at any time (ft <sup>2</sup> - slug)
TSTOP	*	Preset time to stop powered portion of program (sec)
TT		Thrust vector in Earth centered coordinate system (lb)
TTTAB		Inverted thrust time table
TWOPI		$2\pi$
V		Velocity of vehicle (ft/sec)
VA		Velocity of sound (ft/sec)
V2		Cross wind perpendicular to the velocity vector (ft/sec)
V3		Cross wind perpendicular to the velocity vector (ft/sec)
V MACH		Mach number of vehicle
VWX		Wind components in Earth-centered system (ft/sec)
VWY		Wind components in Earth-centered system (ft/sec)
VWZ		Wind components in Earth-centered system (ft/sec)
VX		Velocity vector with wind in Earth-centered system (ft/sec)
WALT	*	Input metro altitude table (ft)

INPUTS

WALTU		Inverted metro altitude table (ft)
WDEN	*	Input metro atmosphere density ( $\text{kgm}/\text{m}^3$ )
WDU		Inverted metro atmosphere density ( $\text{slugs}/\text{ft}^3$ )
WGT		Mass of remaining missile for each stage (slugs)
WGTC		Inverted mass table (slugs)
WGTCI		Mass of fuel for each stage (slugs)
WGTCI		Mass expelled from ignition (slugs)
WGTI		Mass at a given time $\text{WGTI} = \text{WGT} - \text{WGTCI}$ (slugs)
WINDA	*	Local wind azimuth (deg from North, clockwise)
WINDV	*	Local wind velocity (ft/sec)
WMAX		Maximum altitude of metro table (ft)
WPRES	*	Input metro pressure table (millibars)
WPU		Inverted metro pressure table ( $\text{lb}/\text{ft}^2$ )
WTEMP	*	Input metro temperature table ( $^{\circ}\text{C}$ )
WTU		Inverted speed of sound table (ft/sec)
WX		Wind component parallel to Earth-centered X axis (ft/sec)
WY		Wind component parallel to Earth-centered Y axis (ft/sec)
WZ		Wind component parallel to Earth-centered Z axis (ft/sec)
X1X		Variables used in geodetic to geocentric conversion
X1Y		Variables used in geodetic to geocentric conversion
XDIMP		Velocity vector at burnout for impact (ft/sec) +
XIMPA		Position vector at burnout for impact (ft) +
XJ		First harmonic coefficient for oblateness
XMACH		Mach number table for $C_D A/m$ +
XSP		Spin rate at any given time (rad/sec)
XSPT		Integrated spin at any given time (rad)
XVX		Variable used in calculation of PX
Z2		Lines-per-page count for Two Body output +

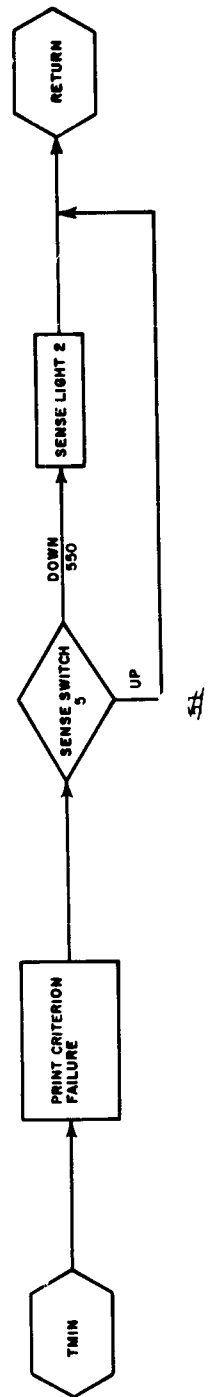
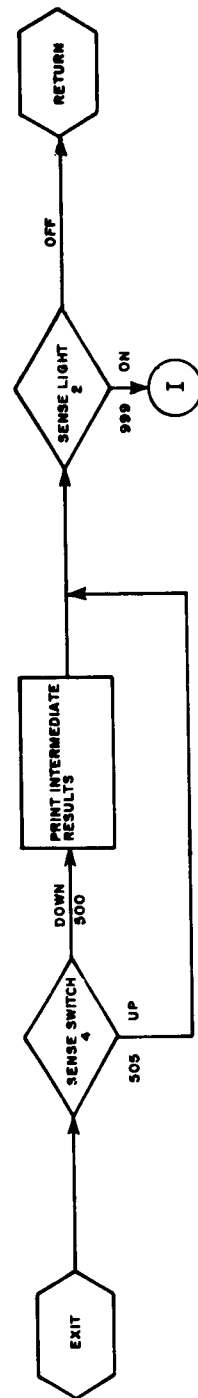
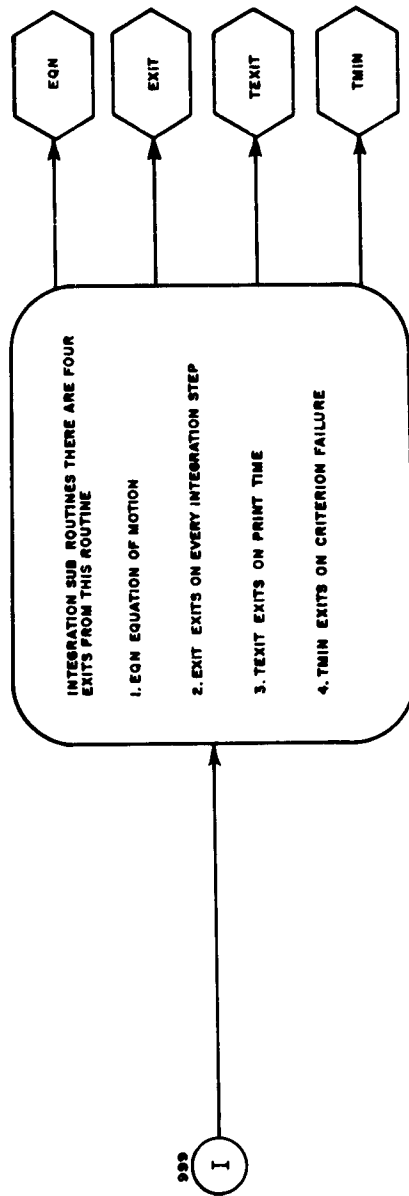
+ Stored in common

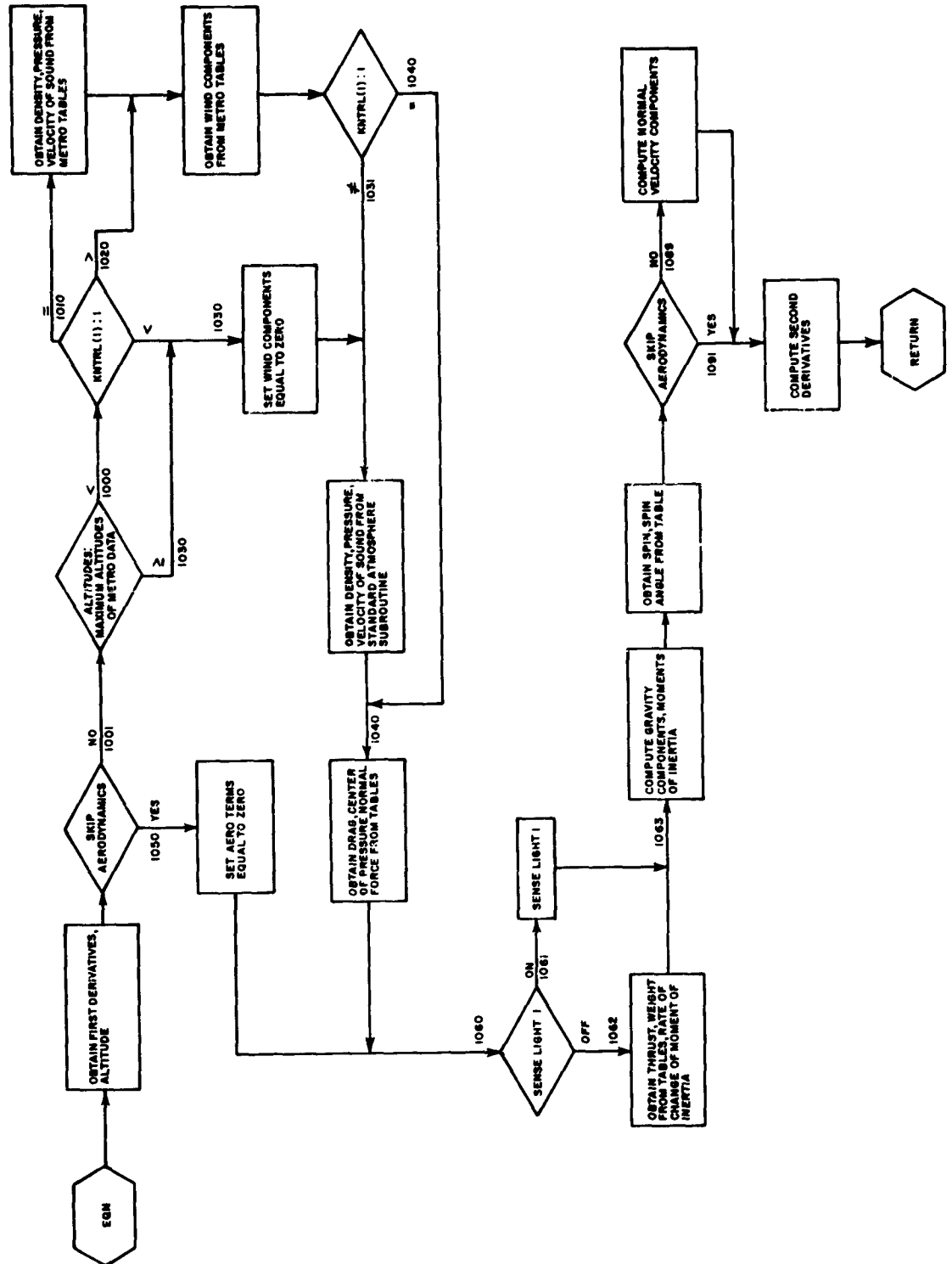
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5. SPURT FLOW DIAGRAMS.

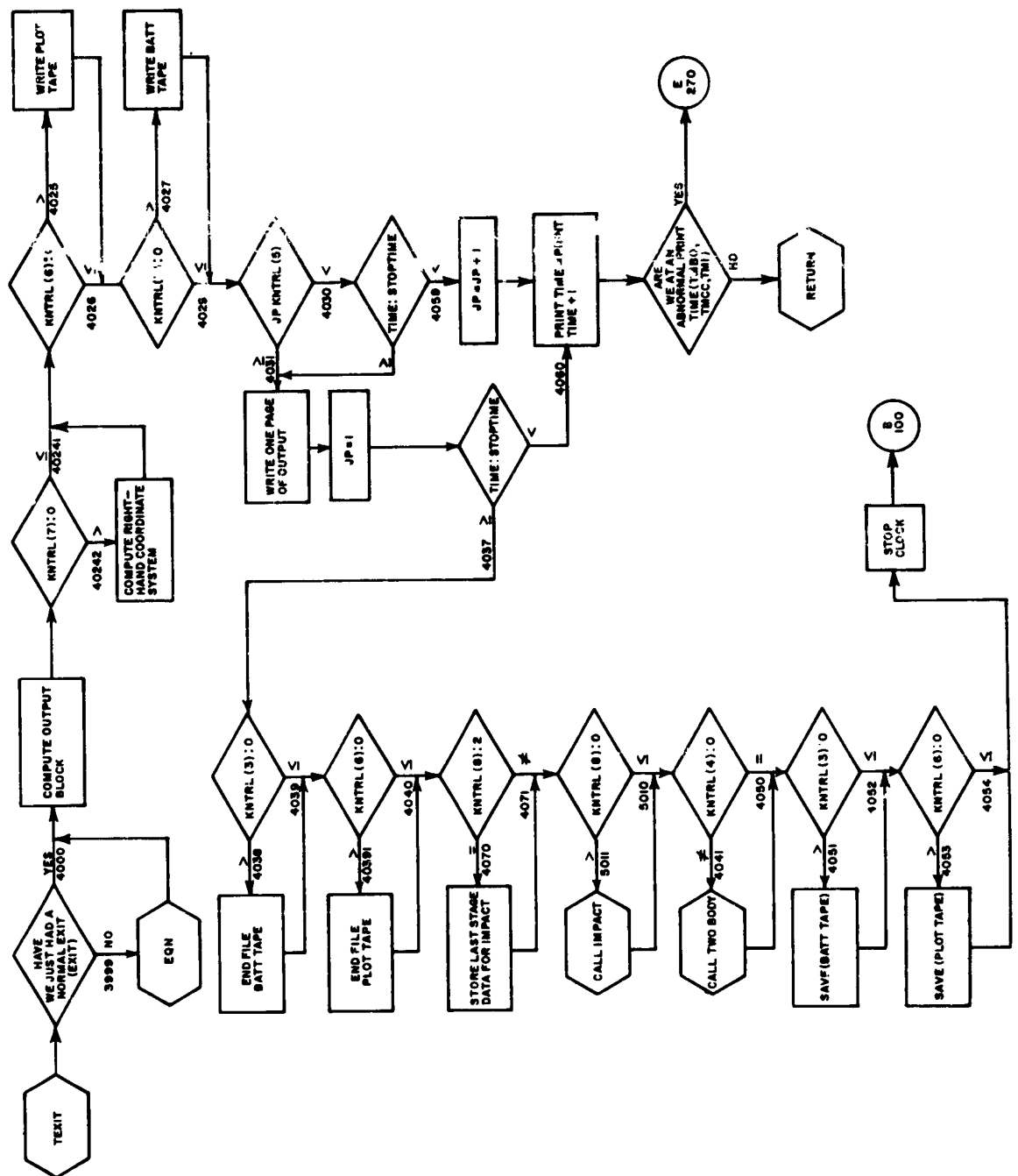












6. SUBROUTINES.

a. Atmosphere subroutine.

The 1962 COESA standard atmosphere is incorporated as a subroutine of SPURT.<sup>15</sup> This atmosphere uses the equations derived for the 1959 ARDC model atmosphere.<sup>16</sup> These equations are

- (1) Molecular-scale temperature

$$T_m = (T_m)_b + L_m (H - H_b)$$

- (2) Pressure - altitude

$$P = P_b \left[ \frac{(T_m)_b}{(T_m)_b + L_m (H - H_b)} \right]^{\frac{GM_o}{R^* L_m}} \quad \text{for } L_m \neq 0$$

$$P = P_b \exp \left[ \frac{-GM_o (H - H_b)}{R^* (T_m)_b} \right] \quad \text{for } L_m = 0$$

- (3) Density - altitude

$$\rho = 3.236598 \times 10^{-4} \frac{P}{T_m}$$

- (4) Speed of sound

$$V_s = 1116.4437 \sqrt{T_m / T_{m_o}}$$

where  $b$  = subscript that refers to the quantity at the base of the constant-gradient layer.

$GM_o / R^*$  = constant of the air gas

$H$  = geopotential altitude in meters

$H_b$  = geopotential altitude in meters at the base of a constant gradient layer

$L_m$  =  $dT_m / dH$  = the gradient of the molecular scale temperature

$P$  = pressure

- $P_b$  = pressure at the base of a particular layer  
 $T_m$  = the molecular scale temperature  
 $(T_m)_b$  = the value of  $T_m$  at the base of a particular layer  
 $V_s$  = speed of sound (ft/sec)  
 $\rho$  = density of the air (slugs/ft<sup>3</sup>)

The skeleton of the COESA is from reference 15 and is given in table 1.

TABLE 1.

## SKELETON OF THE U.S. STANDARD ATMOSPHERE—1962

Defining temperature and molecular weights of the proposed U. S. Standard Atmosphere and computed pressures and densities, where  $z$  = geometric altitude,  $h$  = geopotential altitude,  $T$  = kinetic temperature,  $M$  = mean molecular weight,  $L$  = gradient of molecular scale temperatures =  $dT_m/dh$  (below 79 geop. km) =  $dT_m/dz$  (above 79 geop. km),  $T_m$  = molecular scale temperature =  $(T/M) M_0$ ; and  $M_0$  = sea-level value of  $M$ .

$z$ , km	$h$ , km	$T_m$ , K	$L$ , K/km	$M$	$T$ , K	$P$ , mb	$\rho$ , g/m <sup>3</sup>
0.000	0.000	288.15	-6.5	28.966	288.15	10.1325 / 2*	1.2250 / 3*
11.019	11.000	216.65	0.0	28.966	216.65	2.2632 / 2	3.6392 / 2
20.063	20.000	216.65	1.0	28.966	216.65	5.4747 / 1	8.8033 / 1
32.162	32.000	228.65	2.8	28.966	228.65	8.6798 0	1.3225 / 1
47.350	47.000	270.65	0.0	28.966	270.65	1.1090 0	1.4275 0
52.429	52.000	270.65	-2.0	28.966	270.65	5.8997 - 1	7.5939 - 1
61.591	61.000	252.65	-4.0	28.966	252.65	1.8209 - 1	2.5108 - 1
79.994	79.000	180.65	0.0	28.966	180.65	1.0376 - 2	2.0009 - 2
90.000	88.743	180.65	3.0	28.966	180.65	1.6437 - 3	1.1698 - 3
100.000	98.451	210.65	5.0	28.88	210.02	3.0070 - 4	4.9731 - 4
110.000	108.129	260.65	10.0	28.56	257.00	7.3527 - 5	9.8277 - 5
120.000	117.777	360.65	20.0	28.07	349.49	2.5209 - 5	2.4352 - 5
150.000	146.542	960.65	15.0	26.92	892.79	5.0599 - 6	1.8350 - 6
160.000	156.071	1,110.65	10.0	26.66	1,022.20	3.6929 - 6	1.1584 - 6
170.000	165.572	1,210.65	7.0	26.40	1,103.40	2.7915 - 6	8.0330 - 7
190.000	184.485	1,350.65	5.0	25.85	1,205.40	1.6845 - 6	4.3450 - 7
230.000	221.968	1,550.65	4.0	24.70	1,322.30	6.9572 - 7	1.5631 - 7
300.000	286.478	1,830.65	3.3	22.66	1,432.10	1.8828 - 7	3.5831 - 8
400.000	376.315	2,160.65	2.6	19.94	1,487.40	4.0278 - 8	6.4945 - 8
500.000	463.530	2,420.65	1.7	17.94	1,499.20	1.0949 - 8	1.5758 - 9
600.000	548.235	2,590.65	1.1	16.84	1,508.10	3.4475 - 9	4.6362 - 10
700.000	630.536	2,700.65		16.17	1,507.60	1.1908 - 9	1.5361 - 10

\*Power of 10 by which preceding number must be multiplied.

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This routine must be called by the CODAP symbolic language as follows:

	LDA	ALT
	RTJ	ATMOS
TEM	OCT	
PRES	OCT	
RHO	OCT	
VA	OCT	
	ERR	
	N. R.	

where

ALT	is the altitude in feet
TEM	is the temperature
PRES	is the pressure
RHO	is the density
VA	is the speed of sound
ERR	is the error return
N. R.	is the normal return

b. Subroutine E clock and subroutine S clock.

These subroutines are incorporated to compute the time used by the computer in computing a typical trajectory. These subroutines are written in the CODAP symbolic language and are callable by FORTRAN. The S clock subroutine will initialize the computer clock to zero, and the E clock subroutine will stop the clock and print out the elapsed time in hours, minutes, and seconds.

To use the S clock subroutine:

CALL SCLOCK

To use the E clock subroutine:

CALL ECLOCK(N) where time is printed on tape  
number N.

c. Subroutine GEODED.

The subroutine converts geocentric latitude and radius to geodetic latitude and altitude by use of the following equations.<sup>8</sup>

$$\theta_G = \theta_c + \sin^{-1} \left\{ \frac{a_E}{r} \left[ f \sin 2\theta_c + f^2 \sin 4\theta_c \left( \frac{a_E}{r} - \frac{1}{4} \right) \right] \right\}$$

$$H_G = r - a_E \left[ 1 - f \sin^2 \theta_c - \frac{f^2}{2} \sin^2 2\theta_c \left( \frac{a_E}{r} - \frac{1}{4} \right) \right]$$

where  $a_E$  = equatorial radius of the Earth  
 $f$  = flattening of the Earth  
 $r$  = geocentric position vector  
 $H$  = geodetic altitude  
 $\theta$  = latitude  
 $c$  = refers to geocentric  
 $G$  = refers to geodetic

This subroutine is callable by FORTRAN.

To use:

CALL GEODED (A, B, C, D)

where A is the geocentric latitude (radians)  
 B is the geocentric position vector (feet)  
 C is the geodetic latitude (radians)  
 D is the geodetic altitude (feet)

d. Impact subroutine.

The Impact subroutine is a point mass three-degree-of-freedom trajectory subroutine. This routine is incorporated primarily to compute the trajectories of the "separated" expended stages.

The vector form of the equation of motion is the geocentric position equation derived in section 2 of this report and is

$$\frac{d^2 \vec{R}}{dt^2} = \frac{\vec{\Sigma F}}{M} + 2 \left( \frac{d\vec{R}}{dt} \times \vec{\omega}_E \right) - \vec{\omega}_E \times (\vec{\omega}_E \times \vec{R})$$

The atmosphere subroutine is used along with the oblate Earth described in section 2. The drag parameter ( $C_d A/M$ ) vs. Mach number table for the expended stages are read in the main program and placed in common for use by the Impact subroutine.

The Integration routine will use the same option as the main portion of the program (powered flight) uses. When the altitude is negative, the Impact subroutine will punch a card containing the name, stage number, impact latitude, and longitude. The routine will then terminate and return to the main program. To use the Impact subroutine, control number 8 must be set equal to 1 or to 2.



VARIABLES USED IN IMPACT SUBROUTINE

AB	Not used in impact	*
AC	Not used in impact	*
ACODE	Integration code	+
ACODES	Integration code table	+
AD	Not used in impact	*
AE	Equatorial radius of the Earth	
AG	Not used in impact	*
AH	Not used in impact	*
AI	Not used in impact	*
AK	Not used in impact	*
AL	Block reserved for equivalence	
ALT	Altitude of empty stage	
AM	Not used in impact	*
AN	Not used in impact	*
AN1	Variables used for output parameters	
AN2	Variables used for output parameters	
AO	Not used in impact	*
AP	Not used in impact	*
ARG	Argument used for gravitational computation	
CBLACK	Block reserved for integration	
CDM	$C_D A/M$ - drag parameter	
CHECK	Check used in integration	*
COSRA	Cosine of the range angle	
DELPR	Print time increment	*
DM	Drag over mass parameter	
DRA	Variable used in computing drag	
DRAG	Drag parameter of empty stage	
DUMB1	Not used in impact	*
ERR	Integration error	
FMN	Mach number of empty stage	

---

\* Stored in common

+ Equivalence with AL

GM	Gravitational constant	
GX	X component of gravitational attraction	
GY	Y component of gravitational attraction	
GZ	Z component of gravitational attraction	
H	Variable used for first time increment	
I	Utility index	
ICODE	Code for method of integration	+
II	Utility index	
JP	Page count	
JJ	Number of stage being computed	
J6	Number of lines per page	
K	Utility index	
KAA	Not used in impact	*
KAF	Not used in impact	*
KAZ	Not used in impact	*
N	Number of values in drag table	
NAME	Name stored in common	*
NN	Integer for drag selection	*
NOE	Number of equations for integration	+
NOT	Number of output tape	*
OUT	Dimensioned output variables	
PE	Earth flattening constant	
PI	$\pi = 3.1415927$	
PRTIM1	Print time for integration routine	
R	Length of position vector	
RAD	$= \pi/180. = 1/57.2957795$	
RANGE	Great circle range from launch point	*
R2	Position vector squared	
SIT	Sine of the latitude	
SMAX	Maximum integration step size	+
SMIN	Minimum integration step size	+

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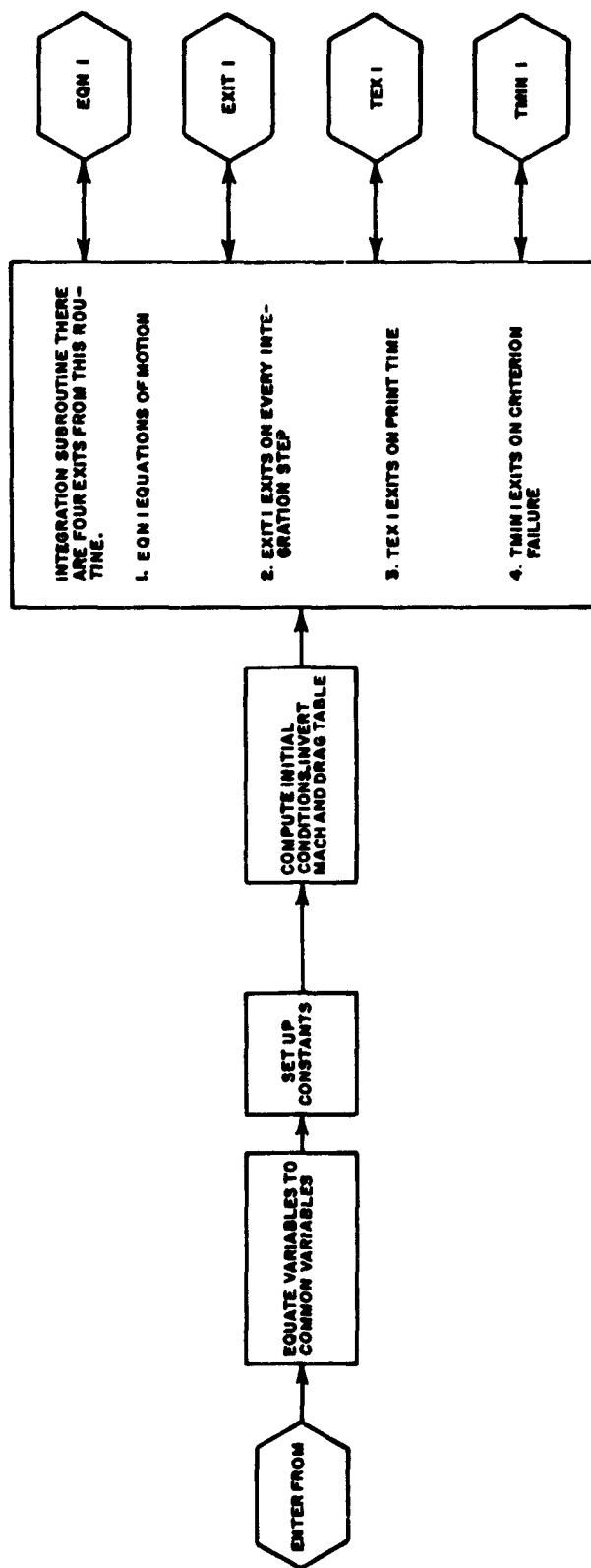
\* Stored in common  
+ Equivalence with AL

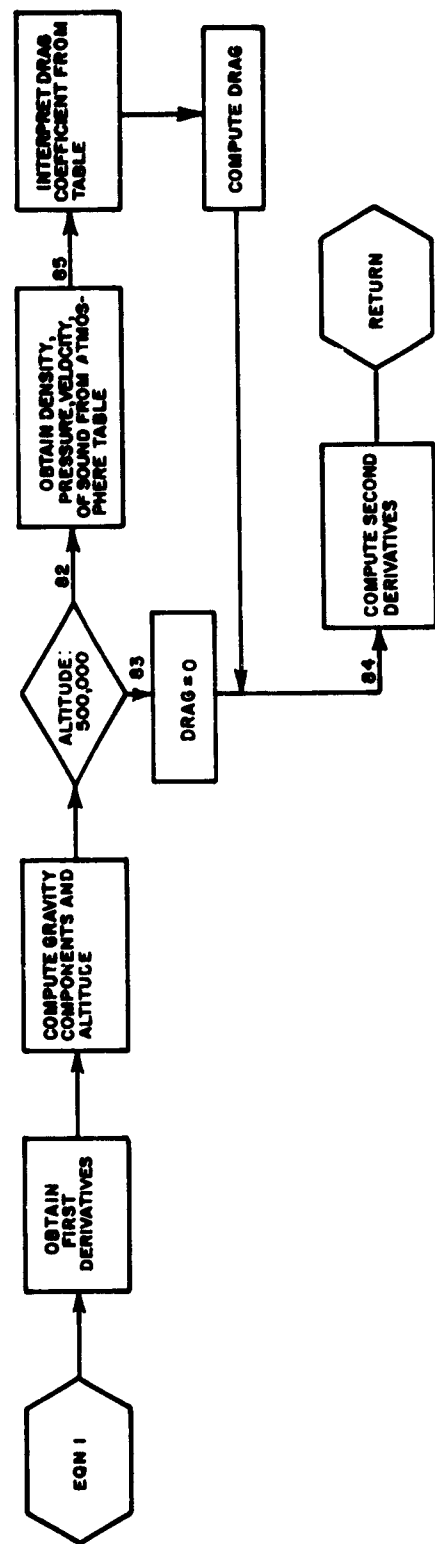
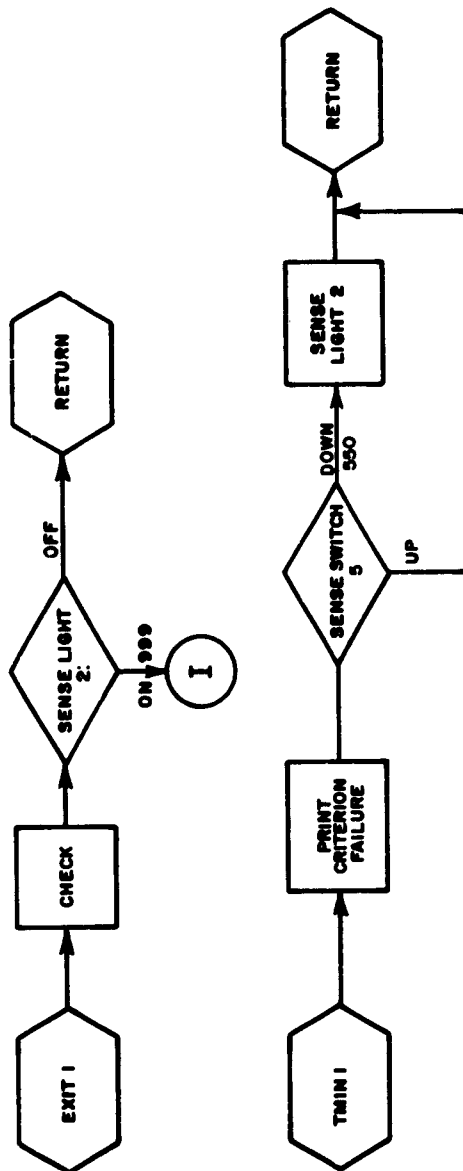
SS	Integration step size	+
T	Time, independent variable	+
TFIMP	Initial time for Impact computation	*
THET	Latitude of position vector	
TRERR	Integration truncation error	+
VA	Velocity of sound	
VD	Vector used for matrix rotation	
VEL	Velocity of empty stage	
W	Rotational velocity of the Earth	
WD	Vector used for matrix rotation	
W2	= W squared	
X	Component of position vector	+
XALT	Altitude from GEOD subroutine	
XD	Velocity vector component	+
XDD	Second derivative of the motion equation	+
XDIMP	Input velocity vector	*
XD1	Velocity used in integration routine	+
XIMPA	Input position vector	*
XK2	Oblateness constant	
XLAT	Geodetic latitude from GEOD subroutine	
XMACH	Mach number table for drag	*
XMN	Stage Mach number table	
XX	Vector for computing range	
XXD	Velocity vector	
Y	Component of position vector	+
YD	Velocity vector component	+
YDD	Second derivative of the motion equation	+
YD1	Velocity used in integration routine	+
Z	Component of position vector	+
ZD	Velocity vector component	+
ZDD	Second derivative of the motion equation	+
ZD1	Velocity used in integration routine	+
Z2	Number of lines per page	*

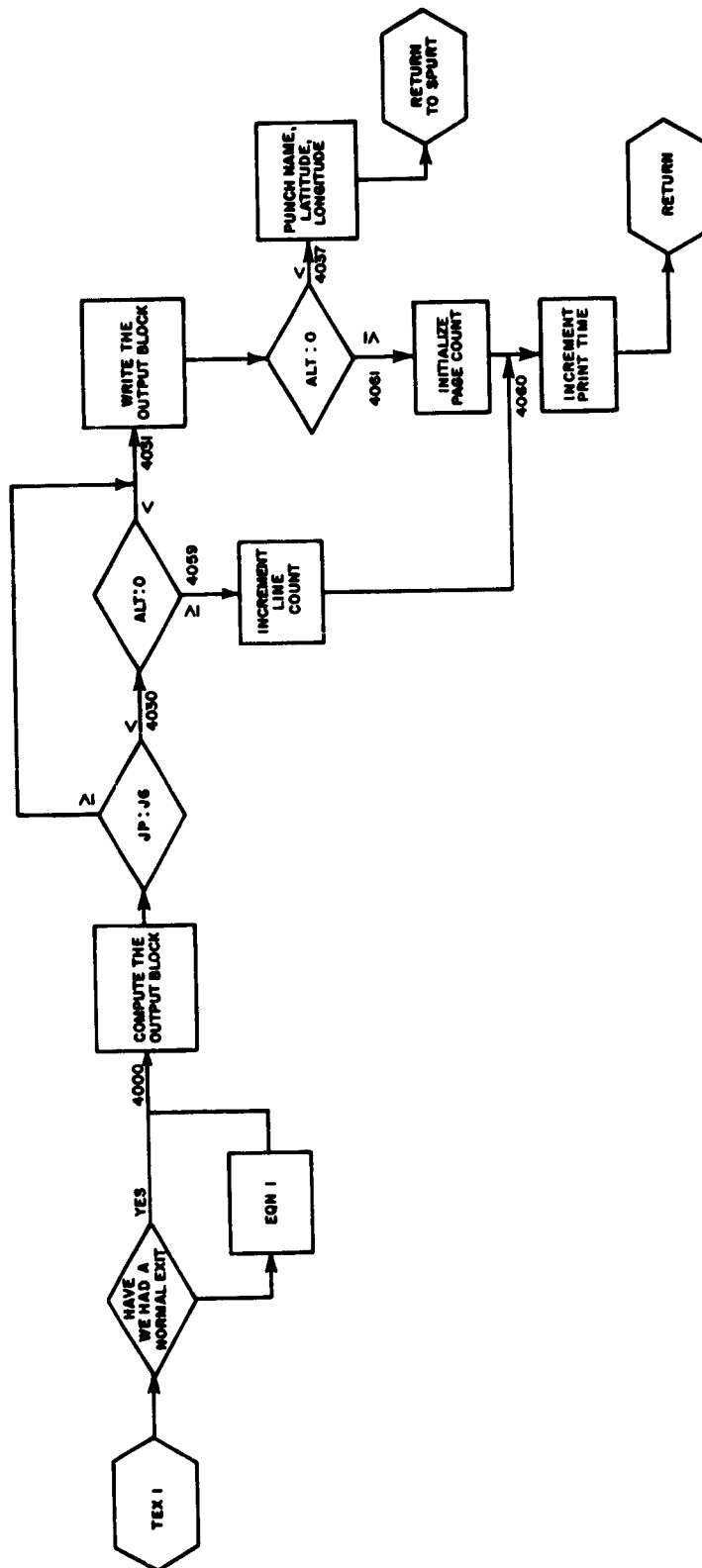
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\* Stored in common

+ Equivalence with AL







e. Integration.

A. IDENTIFICATION

TITLE: Numerical Integration of Ordinary Differential Equations with Error Control

CO-OP ID: D2 CODA NMI 2

PROGRAMMER: Roger Johnson

DATE: February 27, 1961

B. PURPOSE

To integrate a set of N simultaneous first order differential equations of the form:

$$\frac{dx_i}{dt} = f_i(t, x_1, x_2, \dots, x_N), i = (1, 2, \dots, N).$$

The user has the option of either a Runge-Kutta or Adams-Moulton integration scheme. The integration step size,  $\Delta t$ , may be a variable step size under error control or a fixed step size. A print option causes printing of the initial conditions and various parameters before the start of the integration to help define each case. The user also has an option to break into the program at exact specified values of the independent variable t.

C. USAGE

The index registers are saved. No internal checks of arithmetic faults (exponential overflow, underflow, etc.) are made. When the print option is used, the differential equation routine removes the selection of interrupt on arithmetic faults before printing and does a clear arithmetic faults after printing. The differential equations routine should not cause an arithmetic fault unless the  $x_i$ 's exceed the range of the floating point format.

## 1. Calling Sequence -

The calling sequence used to start or restart; if parameters are changed, the integration of a set of differential equation is

<u>Loc</u>	<u>OPN</u>	<u>B</u>	<u>M</u>	<u>OPN</u>	<u>B</u>	<u>M</u>
BETA	SLJ	4	ADAMS	0	P	DERIV
BETA+1	0	0	TEXT	0	0	T
BETA+2	0	0	DATA	0	0	COMMON
BETA+3	0	0	EXIT	0	0	TMIN
BETA+4	ERROR RETURN					

## 2. Parameters -

The upper part of the first instruction of the calling sequence is a return jump command SLJ 4 ADAMS, to the first location of differential equation routine.

If P is unequal to zero the differential equation routine will print the initial conditions and the parameters in the DATA region using the Generalized Listable Output Routine (J5 LMSD OUTPUT). If P is zero no printing takes place.

DATA: starts a block of  $3N+8$  locations that the user sets up with the parameters and variables. The location and description of the parameters and variables are as follows:

DATA contains the integration scheme code which is a fixed point binary integer (binary point at the far right) set up by the user.

- a) If code = 0: The integration scheme is the Runge-Kutta mode with a fixed  $\Delta t$ .
- b) If code = 1: The integration scheme is the Runge-Kutta predictor-corrector mode with a variable  $\Delta t$ .



- c) If code = 2: The integration scheme is the Adams-Moulton mode with a fixed  $\Delta t$ .
- d) If code = 3: The integration scheme is the Adams-Moulton predictor-corrector mode with a variable  $\Delta t$ .
- e) If the code is negative or any binary digit greater than 3, the code is out of range. If this occurs, the AC is set to the binary integer 2 and a jump is made to the error return in BETA+4.

DATA+1 contains the floating point number A used in the variable step mode to prevent unnecessary halving of  $\Delta t$  when  $x_i$  becomes small in magnitude. If A is positive then a positive floating point number  $A_i$  must be determined for each  $x_i$ , ( $i = 1, 2, \dots, N$ ), and be set up by the user in the following locations:

$A_1$  in DATA + 2N + 8

$A_2$  in DATA + 2N + 9

$A_N$  in DATA + 3N + 7

If A is a negative floating point number then the absolute value of A is set in location DATA + 2N + 8 and used for all  $A_i$  ( $i = 1, 2, \dots, N$ ). If A is zero then location DATA + 2N + 8 is set to zero and A is ignored in the halving and doubling option.

DATA+2 contains a positive floating point number E used in the truncation error test in the predictor-corrector variable  $\Delta t$  mode. If E is set to  $10^{-h}$  approximately h significant figure are asked for in the truncation error test. ( $10^{-1} \leq E \leq 10^{-8}$ ) is the suggested range of E.

DATA+3 contains a positive floating point number called MINIMUM DT. If in the truncation error test, after a step  $\Delta t$ , one or more of the variables doesn't meet the convergence test and  $\Delta t$  is less than or equal to MINIMUM DT, the differential

equation routine does a return jump to the users subroutine starting at TMIN. The location of TMIN is given in the calling sequence. The user may stop and do some checking or do an unconditional jump to TMIN to return to the differential equation routine. The differential equation after the return accepts the step  $\Delta t$  ignoring the failure of the convergence test and continues the integration.

DATA+4 contains the positive floating point number MAXIMUM DT. If in the truncation error test all the variables have converged so well that doubling is indicated but  $\Delta T$  is already greater or equal to MAXIMUM DT then  $\Delta T$  is not doubled and the next integration step is still  $\Delta T$ .

DATA+5 contains a positive fixed point integer  $N$ , the number of differential equations to be solved.

DATA+6 contains the floating point number DELTA T. In the fixed  $\Delta T$  mode the whole integration is done with the fixed integration step DELTA T. In the halving and doubling mode the initial trial step is  $\Delta T$  but if the convergence test fails, the initial step will be redone at half  $\Delta T$  and this halving will continue until the convergence test is passed. If the convergence test indicates doubling the next step made will be  $2\Delta T$ . If on entering the differential equation routine DELTA T is zero or negative the AC is set to the binary integer + 1 and an unconditional jump is made to the error return BETA + 4.

DATA+7 is the location of independent variable  $t$ . The user must set up an initial value of  $t$  (a floating point number) that can be negative, zero, or positive. The differential equation will advance  $t$  by some  $\Delta t$  during each integration step.

DATA+8 is the beginning of the  $N$  locations containing the dependent variable  $x_1$  through  $x_N$ . The user sets them up to their initial values at the start of a problem. The differential equation integrates them with step  $\Delta t$  and replaces

them with their new values.

$x_1$  is in location DATA + 8

$x_2$  is in location DATA + 9

. . . . .

$x_N$  is in location DATA + N + 7

DATA + N + 8 is the beginning of the N locations of the derivatives of  $x_i$  or  $f_i(t, \dot{x}_1, x_2, \dots, x_N)$ . The user must code a subroutine starting at location DERIV (given in the calling sequence) to calculate the derivatives of  $x_i$  using the values of  $t, x_1, x_2$ , through  $x_N$  in their DATA locations and then store the derivatives in their locations in the DATA region. As the DERIV subroutine is entered by a return jump from the differential equations routine, an unconditional jump to location DERIV gives the return to the differential equations routine so it can continue the solution. The DATA locations of  $f_i$ , or the derivatives are:

$$\frac{dx_1}{dt} = f_1 \text{ is in location DATA + N + 8}$$

$$\frac{dx_2}{dt} = f_2 \text{ is in location DATA + N + 9}$$

$$\frac{dx_N}{dt} = f_N \text{ is in location DATA + 2N + 7}$$

DATA + 2N + 8 is the beginning of the N locations of  $A_i$ . These are the last locations used in the DATA region. A description of the  $A_i$ 's has already been given under the discussion of DATA + 1.

DERIV is the beginning of a subroutine to be coded by the user. The user's DERIV subroutine function uses the variables  $t$  and  $x_i$  in the DATA regions to calculate the derivatives,  $f_i$ , and stores them in the DATA region. The location of the variables

in the DATA region have been described before, but to repeat:

$t$  is in DATA + 7  
 $x_1$  is in DATA + 8  
 $x_2$  is in DATA + 9  
 . . . . .  
 $x_N$  is in DATA + N + 7  
 $f_1$  is in DATA + N + 8  
 $f_2$  is in DATA + N + 9  
 . . . . .  
 $f_N$  is in DATA + 2N + 7

After calculating and storing the  $f_i$ 's the DERIV routine does an unconditional jump to DERIV which causes a return to the differential equations routine so it can continue the solution. No printing should be done in this routine as the  $x_i$ 's in the DATA region may be just preliminary estimates.

EXIT is the beginning of a subroutine coded by the user to perform printing. When  $t$  and the  $x_i$ 's have been integrated a step  $\Delta t$  and the  $x_i$ 's have satisfied the convergence conditions, the differential equation routine goes to the DERIV subroutine which calculates the derivatives of the new  $x_i$ 's. The differential equation routine then executes a return jump instruction, SLJ 4 EXIT, to the users subroutine for possible printing. The user's subroutine can just bump a counter and do printing just every  $k$  steps or can print at each step. The user can take selected values of  $t$ ,  $x_i$  and the derivatives of  $x_i$  from the DATA region, or calculate functions of these variables and print them or save them for future interpolation.

If the Generalized Listable Output Routine is used the user should remove the selection of interrupt on arithmetic fault before printing as the output routine often causes arithmetic faults not related to computational errors. After printing, a

clear arithmetic faults instruction should be given to clear possible arithmetic faults from the output routine. At the end of the EXIT subroutine the user does an unconditional jump to EXIT to return control to the differential equation routine.

TEXT and T are set up by the user to get points on the solution of the differential equation for specified values of the independent variable  $t$ . The way it works is if the independent variable  $t$  is equal to or greater than the contents of location  $T$ , the differential equation routine will do a return jump to TEXT with the contents of location  $T$  minus the independent variable  $t$  in the AC. In using this feature the contents of location  $T$  should be always set greater than the independent variable  $t$ . Because if the next integration step will make  $t$  larger than the contents of location  $T$  the differential equation routine does a special Runge-Kutta integration step with a value of  $\Delta t$  to advance  $t$  and calculate  $x_1$  and the derivatives of  $x_1$ .  $\Delta t$  is such that  $t$  is equal to the contents of location  $T$ . After this special  $\Delta t$  step the old  $\Delta t$  is restored and the differential equation routine does a return jump, SLJ 4 TEXT, to the user's TEXT subroutine and as  $t$  equals the contents of location  $T$  the AC is zero. The user's subroutine, TEXT, may do printing, keeping in mind the suggestions given in the EXIT subroutine. It should then advance the contents of  $T$  to the next break point. To exit, an unconditional jump to TEXT returns to the differential equation routine.

If the contents of  $T$  are less than the independent variable  $t$  because of improper updating or some other oversight, the differential equation routine still executes the instruction, SLJ 4 TEXT, but with the AC minus to show that the break point has been passed in  $t$ . The break point features of the differential equation routine will be ignored if  $T$  or the contents of location  $T$  are set to zero, then no jump to TEXT will ever occur. If both  $T$  and the contents of

location  $T$  are nonzero, but  $TEXT$  is zero, the differential equations routine will go to its error return with a fixed integer -1 in the AC because no subroutine can start at zero.

If, when  $t$  equals the contents of location  $T$  the formulas to compute the derivatives of  $x$  are changed or it is wished to change some of the parameters in the DATA region, the solution of the differential equation must be restarted by either going to a new calling sequence or back to the start of the old calling sequence.

If  $t$  is at the end of a case, the next case can be set up and the differential equation routine restarted with a calling sequence. Only in the user's  $EXIT$  and  $TEXT$  subroutines can the values of the variables be trusted either for printing or restarting the solution by going to a calling sequence.

BETA + 4 is the error return for the differential equation routine. An unconditional jump is made to  $BETA + 4$ , with a fixed point binary integer in the AC which tells the type of error, if a parameter is obviously bad. To correct this, the bad parameter should be changed and the case rerun. When at location  $BETA + 4$ , if:

AC is 0: then  $N$ , the number of equations to be solved, (location  $DATA + 5$ ) has been set to zero, a negative number, or is greater than 200.

AC is +1: then  $\Delta t$  (location  $DATA + 6$ ) is either zero, negative, or not in normalized floating point form.

AC is +2: then the integration code in location  $DATA$  is either negative or greater than + 3.

AC is -1: Both  $T$  and the contents of  $T$  are nonzero, but  $TEXT$  is zero. This means the exact  $t$  break feature is being used but the users subroutine,  $TEXT$ , starts at location zero. This is invalid.

COMMON is a block of  $14N + 5$  locations starting at COMMON that the user must reserve for the differential equation routine. When command has been transferred to the users EXIT subroutine the first  $N$  location of COMMON contain the predictor values of the new  $x_i$ . The truncation error of the last step for the variable  $x_i$  is about  $1/14 (x_{p,i} - x_i)$  in magnitude. The locations of the variables  $x_{p,i}$  and  $x_i$  are:

$x_{p,1}$  is in location COMMON  
 $x_{p,2}$  is in location COMMON + 1  
 . . . . .  
 $x_{p,N}$  is in location COMMON + N-1

If the user needs the values of  $t$  and the variables  $x_i$  of the last step for interpolation they can be found in the locations:

$t$  in COMMON + N  
 $x_1$  in COMMON + N + 1  
 $x_2$  in COMMON + N + 2  
 . . . . .  
 $x_N$  in COMMON + 2N

As already described in the writeup, the current  $x_i$ 's are in the DATA locations:

$x_1$  in DATA + 8  
 $x_2$  in DATA + 9  
 . . . . .  
 $x_N$  in DATA + N + 7

TMIN is the first location of a subroutine coded by the user that the differential equation goes to if the convergence test has failed and  $\Delta t$  is less than or equal to MINIMUM DT.

Its use is discussed later in this writeup.

3. SPACE REQUIRED - is about 700 locations not including the 610 locations of the Generalized Listable Output routine.
4. TEMPORARY STORAGE - none. The two blocks of storage DATA (3N+ 8 locations) and COMMON (14 N+ 5 locations) cannot be used by the programmer to store numbers.
7. ERROR STOPS - the error return is described before. It is in location BETA + 4 in the calling sequence.
9. PRINT INFORMATION - If the print option is used the "General Listable Output Routine" must be in the computer and its symbol location, OUTPUT, in 70412B. At the head of the assembly is the card

OUTPUT EQU                      70412B

The symbolic location ADTAPE, in the differential equation routine determines the channel number of the output tape, the tape number, and 1607 number. The print option sets up words for the write BCD tape calling sequence by using ADTAPE after inserting the proper carriage control in the lower address. Therefore, to change the channel number, tape number, or 1607 number, just modify the contents of ADTAPE in the differential equation routine. At present:

CN or channel number of the output tape is 4

TN or tape number of the output tape is 4

UN or 1607 number is 2

#### D. METHOD

To compute the change of  $x_i$  with the integration step  $\Delta t$  the fourth order Runge-Kutta method gives the formulas:



$$k_{1,i} = (\Delta t) f_i(t, x_1, x_2, \dots, x_n)$$

$$k_{2,i} = (\Delta t) f_i(t + 1/2 \Delta t, x_1 + 1/2 k_{1,1}, \dots, x_n + 1/2 k_{1,n})$$

$$k_{3,i} = (\Delta t) f_i(t + 1/2 \Delta t, x_1 + 1/2 k_{2,1}, \dots, x_n + 1/2 k_{2,n})$$

$$k_{4,i} = (\Delta t) f_i(t + \Delta t, x_1 + k_{3,1}, \dots, x_n + k_{3,n})$$

$$\Delta x_i = \frac{1}{6} (k_{1,i} + 2k_{2,i} + 2k_{3,i} + k_{4,i})$$

The calculation:  $x_i^C(t + \Delta t) = x_i(t) + \Delta x_i$  is done in double precision to help control rounding errors. All other calculations are performed in single precision. Programers at STL didn't feel the Gill or other modifications to control rounding errors to be of much value so the standard Runge-Kutta method was programed.

The Adams-Moulton method requires the derivatives of  $x_i$  three equal steps behind. To start the Adams-Moulton integration, several Runge-Kutta integrations are required. The Adams-Moulton predictor-corrector formulas are:

$$x_{i,k+1}^p = x_{i,k} + \frac{\Delta t}{24} (55 f_{i,k} - 59 f_{i,k-1} + 37 f_{i,k-2} - 9 f_{i,k-3})$$

$$\Delta x_{i,k} = \frac{\Delta t}{24} (9 f_{i,k+1}^p + 19 f_{i,k} - 5 f_{i,k-1} + f_{i,k-2})$$

Again the calculation  $x_{i,k+1}^C = x_{i,k+1} + \Delta x_{i,k}$  is done in double precision to help control rounding errors.

Both Adams-Moulton and Runge-Kutta have error terms of the order  $(\Delta t)^5$  but the Runge-Kutta step requires four references to the derivatives against just two references in an Adams-Moulton step; so for most problems the Adams-Moulton integration method is to be preferred.

In integrating a differential equation numerically, it is replaced with a difference equation and we solve this difference equation. If the integration steps used are small and an integration scheme like

Adams-Moulton or Runge-Kutta which have favorable stability properties is used, the solution of the difference equation is usually close to the solution of the differential equation. However, if some of the variables  $x_i$  given by the differential equation routine have either odd oscillations or increase very rapidly with the physical problem not suggesting such behavior, the solution is probably unstable and greatly in error. In long problems with many integration steps a slight instability will cause the solution to slowly drift from the true solution of the differential equation. An unstable solution can often be made stable by integrating with a smaller step  $\Delta t$  or by forcing a smaller step by asking for more accuracy in the predictor-corrector mode. The reason for this is that different step sizes change the parameters relating to the stability of the difference equation. Therefore, if two solutions with different step sizes are similar and close together, both solutions can be accepted as correct. The routine also makes it easy to do checking by running the same case over using both the Runge-Kutta and Adams-Moulton integration methods. Usually the truncation error requirements in solving a set of differential equations dictate integration steps sufficiently small to insure stability.

#### HALFING AND DOUBLING MODE

When doing an Adams-Moulton integration step  $\Delta t$  to advance the dependent variables from  $x_i(t)$  to  $x_i(t + \Delta t)$  for each variable, we first calculate the predicted value  $x_i^P(t + \Delta t)$  for each variable. The users DERIV routine is used to calculate the predictor derivatives,  $f_i(t + \Delta t, x_1^P(t + \Delta t), \dots, x_n^P(t + \Delta t))$ . Using the predicted derivatives the step is finished by calculating  $x_i^C(t + \Delta t)$  the corrector. An estimate of the magnitude of the truncation error of each variable in the last step is:

$$\frac{1}{14} \left| x_i^C(t + \Delta t) - x_i^P(t + \Delta t) \right|$$

To get an estimate of the truncation error in the Runge-Kutta mode

we first do a Runge-Kutta integration step of  $2\Delta t$  to get the predictor,  $x_i^P(t + 2\Delta t)$ . Then restarting with the variables  $x_i(t)$  we do two Runge-Kutta integration steps each of length  $\Delta t$  to get the corrector,  $x_i(t + 2\Delta t)$ . An estimate of the magnitude of the truncation error for each variable in the step from  $x_i(t)$  to  $x_i(t + 2\Delta t)$  is:

$$\frac{1}{15} \left| x_i^C(t + 2\Delta t) - x_i^P(t + 2\Delta t) \right| .$$

Going from  $x_i(t)$  to  $x_i(t + 2\Delta t)$  requires 4 references to the derivatives using the Adams-Moulton integration scheme and 12 references to the derivatives using the Runge-Kutta scheme.

The convergence test uses the parameters  $A_i$  and  $E$  described before in the writeup of the DATA region. Using the predictor,  $x_i^P$ , and corrector,  $x_i^C$ , of each variable from the last integration step (for either the Runge-Kutta or Adams-Moulton method) if the inequality:

$$\frac{\left| x_i^C - x_i^P \right|}{\max \left[ A_i, \left| x_i^P \right|, \left| x_i^C \right| \right]} < E$$

is satisfied for all  $i$  ( $i = 1, 2, \dots, N$ ) then we say the convergence test has been passed and the results of the last integration step will be accepted by the differential equation routine. If the inequality doesn't hold for any one of the  $i$ , then the convergence test has failed and the results of the last integration step are not accepted. If  $E$  in the above inequality is replaced by  $E/M$ , "where  $M$  is in symbolic location  $ADC + 2$  and set to the value 20 in floating point format", and the above inequality is still satisfied for every  $i$ , we say the convergence test to double has passed. Again, if when  $E$  is replaced by  $E/M$  in the above inequality and the inequality doesn't hold for any one of the  $i$  we say the convergence test to double has failed.

If the convergence test is satisfied but the convergence test to double has failed, then the integration step  $\Delta t$  is left unchanged and

the differential equation routine goes to users subroutine EXIT for possible printing. Upon return to the differential equation routine it integrates the variables again by the same step  $\Delta t$ .

If the convergence test to double is satisfied the differential routine goes to the users subroutine EXIT for possible printing of the accepted  $x_i^C$  but on return a set up may be made to make the next integration step  $2\Delta t$ . Even though the convergence test to double is satisfied, sometimes  $\Delta t$  is not doubled. For instance, if  $\Delta t$  is already greater than or equal to the number, MAXIMUM DT in DATA + 4, then  $\Delta t$  is not doubled. If  $\Delta t$  was halved within the last four steps in the Runge-Kutta mode, controlled by the fixed point binary number 4 in symbolic location ADC, no doubling of  $\Delta t$  is done or if  $\Delta t$  was halved within the last six steps in the Adams-Moulton mode, controlled by the fixed point binary number 6 in symbolic location ADC + 1, no doubling is done. The above delays are inserted to save machine time that might be wasted in halving and doubling oscillations.

In the Runge-Kutta mode doubling can take place every integration step but in the Adams-Moulton mode sufficient delays may not exist to do the next integration step at  $2\Delta t$ . To integrate with the next step  $2\Delta t$ ; first  $x_i(t + \Delta t)$  becomes  $x_i(t^*)$ , where  $t^* = t + \Delta t$  the advanced independent variable, and  $\dot{x}_i(t - 5\Delta t)$  becomes  $\dot{x}_i(t^* - 3(2\Delta t))$ . Therefore, after any halving or doubling operation in the Adams-Moulton mode  $\Delta t$  cannot be doubled until 3 integration steps of the same length are made to get the delays needed for the next integration step  $2\Delta t$ .

If the convergence test has failed  $\Delta t$  is usually halved. The only case where  $\Delta t$  is not halved, is if  $\Delta t$  is less than or equal to the floating point number, MINIMUM DT set up by the user in location DATA + 3. As described before in this writeup then  $\Delta t$  is supposed to half but it is already less than or equal to MINIMUM DT. If a return jump is made to the users subroutine TMIN and if the user returns to the differential equation routine by an unconditional jump to TMIN,

then  $x_i(t + \Delta t)$  is accepted and the next integration step will also be  $\Delta t$ .

If the convergence test has failed and  $\Delta t$  is greater than MINIMUM DT the  $x_i(t + \Delta t)$  in the Adams-Moulton mode and  $x_i(t + 2\Delta t)$  in the Runge-Kutta mode are not accepted and the differential equation routine goes back to variables  $x_i(t)$  and the next integration step is  $.5\Delta t$ . In the Runge-Kutta mode the differential equation routine restores the old  $x_i(t)$  and its derivatives it had saved in COMMON storage and takes the saved result of the first RK step  $x_i(t + \Delta t)$  and makes it the new predictor. It then enters the RK predictor-corrector mode at the point it computes the two Runge-Kutta steps, now each of length  $.5\Delta t$  to get the corrector.

In the Adams-Moulton mode after  $\Delta t$  is halved the derivatives of  $x_i(t - .5\Delta t)$  and  $x_i(t - 1.5\Delta t)$  are needed for the next integration step of length  $.5\Delta t$ . Using  $\dot{x}_i(t)$ ,  $\dot{x}_i(t - \Delta t)$ ,  $\dot{x}_i(t - 2\Delta t)$ ,  $\dot{x}_i(t - 3\Delta t)$ , and  $\dot{x}_i(t - 4\Delta t)$  in the five point interpolation formula of Lagrange;  $\dot{x}_i(t - .5\Delta t)$  and  $\dot{x}_i(t - 1.5\Delta t)$  are computed. The five point formula was used as its error term is of the same order  $(\Delta t)^5$  as the error terms in the Runge-Kutta and Adams-Moulton integration methods with integration steps of length  $\Delta t$ . After interpolation, the routine restores  $x_i(t)$  and returns to the Adams-Moulton mode in the routine to the part where the Adams-Moulton step will be done over with the new step  $.5\Delta t$ . As referred to before, no doubling of  $\Delta t$  will take place after a halving in Adams-Moulton mode until sufficient delays exist for doubling  $\Delta t$ . Tags were set to delay 4 steps in the Runge-Kutta mode and 6 steps in the Adams-Moulton mode to save machine time.

The five point Lagrange Interpolation Formulas came from page 118 of "Introduction to Numerical Analysis" by F. Hildebrand. In the same book can be found the formulas of Runge-Kutta on page 237 and Adams-Moulton on page 200.

SETTING PARAMETERS

In solving a set of differential equations we are interested in keeping the error at all points of the solution within certain specified limits. However, by setting the parameters  $A_i$  and  $E$ , the truncation error is only controlled for each single step in the predictor-corrector mode. The total error is a combination of the truncation and rounding errors of many steps. The truncation error per step for the variable  $x_i$  is less than  $\frac{1}{14} E x_i$  if the absolute value of  $x_i$  is greater than the value of  $A_i$  or less than  $\frac{1}{14} E A_i$  if  $A_i$  is larger of the two. As most problems are only a few thousand integration steps and are well behaved, the truncation error per step is suggested to be set to one fiftieth of the total error allowed.

Two examples for setting  $A$  and  $E$  are:

1) Integrate  $\dot{x} = \cos t$  with truncation error per step less than  $10^{-5}$ . As the truncation error requirement is independent of the magnitude of the variable  $x$ , set  $A$  to 1 which is equal to or greater than the absolute value of  $x$  at any time.

As  $\text{MAX} (A, |x_i^c|, |x_i^p|) = 1$ . set  $E = 14(10^{-5})$ .

2) Integrate  $\dot{x} = e^t$ , where  $x(0) = 1$ , with truncation error per step less than  $10^{-5}$ . As the truncation error requirement per step is some proportion of the variable  $x$  set  $A$  to zero. As  $\text{MAX} (A, |x_i^c|, |x_i^p|) = x_i^c$  set  $E = 14(10^{-5})$ . If when integrating the last case to  $t_f$  seconds we set  $A = e^{t_f}$ ; as  $\text{MAX} (A, |x_i^c|, |x_i^p|) = A = e^{t_f}$  we would have large  $\Delta t$  steps at the beginning resulting in great relative errors that would be carried through the whole solution.

To avoid unnecessary halving of  $\Delta t$ ; each value of  $A_i$  should be set slightly less than the average magnitude of its corresponding variable  $x_i$ , but, if the solution tends to be inaccurate or unstable when some of the variables are small the value of  $A_i$  corresponding to these variables must be decreased.

When trying to see the effects of the range of a parameter to check for the total error of an integration; change the parameter at least by a factor of 10 to make sure it reruns with a different step size and change  $E$  or some of the  $A_i$ 's only to isolate the effect. When running a problem the user is not too familiar with; a typical case, or the extreme cases plus a few middle cases, should be run with different values of the parameters  $A_i$  and  $E$ . Then after comparing the solutions of the same case which has been run with different parameters  $A_i$  and  $E$ , run the rest of the cases with the values of the parameters  $A_i$  and  $E$  that gave satisfactory total error bounds. Sometimes when  $E$  is decreased the greater number of integration steps can increase the total rounding error until it is so much greater than the total truncation error that the solution with the smaller  $E$  has greater total error than the solution with the larger  $E$ ; however, as the variables  $x_i$  are accumulated in double precision and since the Control Data computer has a 36 bit mantissa then only when  $E$  becomes less than  $10^{-8}$  are the total errors likely to increase when  $E$  is further decreased.

When solving some problems a part of the solution may have very small truncation errors; then  $\Delta t$  can become so large that the solution becomes unstable. The parameter MAXIMUM DT, in DATA + 4, is used to prevent  $\Delta t$  from becoming too large. As an example, when a trajectory was being calculated in a region of very thin air, integration steps of 15 seconds passed the truncation error test but the solution soon became unstable and did false large oscillations. By setting a limit on the integration step to a few seconds the rerun solution was stable.

#### BREAK POINTS

As described before in the writeup the contents of T and TEXTIT can be set up by the user to get points of the solution of the differential equation for specified values of the independent variable  $t$ .

One use of this feature is to do printing every  $h$  seconds of  $t$ . Initially, the contents of  $T$  are set to  $h$ . Then the  $k$ -th time the differential equation routine does a return jump to the user's TEXTIT subroutine the user prints the solution, where  $t = kh$ , then bumps location  $T$  by  $h$  and does a return jump to the differential equation routine so it can calculate to the next  $t$  break,  $t = (k+1)h$ .

Another use of this feature is when starting a solution, if some of the variables are initially equal to zero, a different set of parameters is required for a small value of  $t$ . At the start of the problem the contents of  $T$  are set to a value  $t_p$ , slightly greater than the initial value of  $t$  and the starting parameters are set up. When  $t$  equals  $t_p$  the users TEXTIT subroutine is used to modify the parameters.

When just the contents of  $T$  or any of the convergence parameters like  $E$ ,  $A_i$ , MINIMUM DT, or MAXIMUM DT are changed in the middle of a case by a user's subroutine the users normal return, which is an unconditional jump to the beginning of his subroutine may be used in returning to the differential equation routine. However, almost any other kind of change in a users subroutine requires a jump back to the beginning of the calling sequence or a new calling sequence. Such changes by the user requiring a return to a calling sequence are: if the calling sequence is to be modified, either  $N$ , code, or  $\Delta t$  is changed, changes of the variable  $t$  or  $x_i$ . This is necessary because parameter setups from the calling sequence are done only at the beginning of the routine and the logic of the differential equation routine depends on the code to tell where it has stored the variables and delays of the derivatives of  $x_i$  in COMMON. The delays are also assumed to be integral multiples of the present  $\Delta t$  in DATA + 6.

To return to examples in the use of the time interrupt feature. One way to start solutions where some of the initial values of the variables  $x_i$  are zero is to run in the fixed  $\Delta t$  mode with a small



$\Delta t$  until the solution is started. Then, after the solution is assumed started at time  $t_p$  change to the predictor-corrector mode by modifying the code, and restart the solution at  $t = t_p$  by entering a new calling sequence. At the same time TEXTIT could be changed to a new TEXTIT subroutine that does printing. To accurately simulate a missile in the velocity range of zero to a few hundred feet per second when it is under the influence of torques that change its direction (like crosswinds or controllers) requires small integration steps of the order .01 second or large errors in the orientation of the missile occur from the integration process.

The last use of time interrupt discussed is when there are discontinuities of the variables or their derivatives at specified times. An example is when a rocket engine burns out at  $t_p$ . By setting  $t_p$  in location T then the user in his TEXTIT routine must modify his DERIV routine to omit the thrust term and the change the drag calculations. As such changes cause discontinuities in the derivatives of  $x_i$  the differential equation routine should be restarted by going to a new calling sequence.

Another case similar to the above is in the staging of missiles, there should be a break when a missile separates into two sections, a break at the end of free flight of the second stage, and at the time of burnout of the rocket engine; whose firing terminated the free flight of the second stage. When controllers are turned on and off they can cause discontinuities in the derivatives of the variables  $x_i$  or even in  $x_i$  if the controllers are impulse functions, so the differential equation routine should be restarted by entering a calling sequence at such times. When interpolating through nonsmooth functions with discontinuous derivatives to maintain accuracy the differential equation routine halves  $\Delta t$  many times using a great deal of machine time; it may also cause an exit to the users TMIN subroutine because  $\Delta t$  is less than or equal to MINIMUM DT and the error requirements are not satisfied. In runs with controllers even though the errors in interpolating variables through nonsmooth functions may be small in terms of the magnitudes of the variables

$x_i$ , as the error term driving the controller is a function of the difference of some ideal  $x_i$  and actual  $x_i$ , this error may jump several hundred percent making the rest of the controller or guidance simulation useless.

#### USERS INTERPOLATIONS

The user may require other breaks than time breaks. He may wish to print the output every 1000 feet of altitude or turn on or off a controller when the error, which is a function of the  $x_i$ 's, becomes equal to some critical value. To do this the user could save several values of the functions at different times and for some interpolation formulas also the derivatives of the functions and do an inverse interpolation to find the time when the function of the error reached the critical point. The interpolation formula used should have an error term of the order  $(\Delta t)^5$  to have about the same accuracy as the differential equation routine.

After the time of the break  $t$  (break), is known by inverse interpolation the user can interpolate for all the other variables, if he has saved enough delays, then change the method of calculating the derivatives to include the new status of the controller and re-start the differential equation routine by going to a calling sequence. If the user hasn't saved enough delays of the variables to do sufficiently accurate interpolation, he could let the differential equation routine do the interpolation. The user is in his EXIT routine, the time of break is between the current  $t$  in DATA + 7 and the  $t$  that was in DATA + 7 at the time of the last entry into the EXIT routine as the break occurred between steps. The user first moves the old  $t$  and the old variable  $x_i$  from COMMON in this way:

```
old t from COMMON + N to t in DATA + 7
old  $x_1$  from COMMON + N + 1 to  $x_1$  in DATA + 8
. . . . .
old  $x_n$  from COMMON + 2N to  $x_n$  in DATA + N + 7
```

Then set the new  $\Delta T = t(\text{BREAK}) - t(\text{old})$  in DATA + 6 and change the code to zero, by setting zero in location DATA, and go to a calling sequence to restart the differential equations routine. The first return to users EXIT will be with  $t = t(\text{old})$  but in the second return to EXIT the variables and  $x_i$  will be advanced to the break point. The user then makes the necessary changes in his DERIV subroutine that happen at the break point, restores the old code and  $\Delta t$  and goes to a calling sequence to restart the differential equation.

The user shouldn't turn on the controller or ignite the rocket motor in the simulations until he has all his variables interpolated at the break time or he will have serious errors in his interpolations because of the nonsmooth functions introduced. If the user hasn't saved delays of the function that determines the break point he could calculate the old value of the function from  $t$  and the variables  $x_i$  in COMMON + N through COMMON + 2N and do a linear interpolation for the break point. Then advance  $t$  and the variables  $x_i$  to the estimated break from linear interpolation by a Runge-Kutta step as before. Then the estimated  $t$  break from linear interpolation and the new variable  $x_i$  with a new interpolation can be used to get a better estimate of the break time and this process repeated until the break point is calculated to sufficient accuracy. The user should note the process will iterate closer and closer to the break point but may not go past it so the exit from above process should be done when an estimate of the  $t(\text{break})$  is close to the actual  $t(\text{break})$ .

#### TMIN SUBROUTINE

If in the predictor-corrector mode one of the variables has failed the convergence test and  $\Delta T$  is less than or equal to MINIMUM DT the differential equation routine does a return jump to the users subroutine TMIN.

If the user has a problem he knows can be integrated to sufficient accuracy using a step,  $\Delta T$ , he can set MINIMUM DT to  $\Delta T$  and in the TMIN subroutine do an unconditional jump to TMIN

to cause the return to the differential equation. The differential equation routine accepts the last step integrated with the last  $\Delta T$  and also does the next step with the same  $\Delta T$ . Therefore, machine time is not wasted with a  $\Delta T$  much smaller than  $\Delta T$  and if in other regions of the problem, where truncation error is small  $\Delta T$  may be much larger than  $\Delta T$ . The regions of small  $\Delta T$  could be caused by slight discontinuities in curve fits of the empirical data that determine the coefficients of the differential equation or the values of  $A_i$  are too small.

If the user has a problem that isn't too familiar MINIMUM DT should be set very small. Then the TMIN subroutine should stop the problem so it can be examined. The user may find derivatives changing rapidly or which are very large because they are not calculated correctly or an unnoticed singular point is making a derivative infinite. Also the user may be changing the variables because of program errors, or if the differential equation has impulse functions the variables were changed but the differential equation routine wasn't restarted. If the value of MINIMUM DT was only a factor of 10 smaller than the  $\Delta T$  that runs most of the problem accurately, an exit to TMIN could mean, discontinuous derivatives being integrated through a point where the differential equation routine should have been restarted. Another possibility with MINIMUM DT a factor of 10 smaller than the standard DT, if in TMIN when some of the variables are small; the values of some of the  $A_i$  could be increased and the problem reran with both a smaller MINIMUM DT and the old  $A_i$ 's and the old MINIMUM DT and the larger  $A_i$ 's and the solution compared to see if the shorter machine time in running with larger  $A_i$ 's gives sufficiently accurate solutions.

#### RUNNING A PROBLEM IN REVERSE

If a user wishes to run a problem backwards in time, as  $\Delta t$  cannot be made negative in this routine, the method described below must be used.

Given the standard set of  $N$  simultaneous differential equations to be integrated we have:

$$\frac{dx_i}{dt} = f_i \left[ t, x_1(t), x_2(t), \dots, x_N(t) \right]$$

Let the initial value of  $t$  be  $t_0$  and its final value be  $t_f$ . To run this same set of differential equations backwards we make the transformation  $T = t_f - t$ , to the new independent variable  $T$ . Let the initial value of  $T$  be 0, then  $t = t_f$  and the initial values of the variables  $x_i$  is  $x_i(t_f)$ . Then the final value of  $T$  is  $t_f - t_0$  and this corresponds to  $t = t_0$  and the final value of the variables  $x_i$  is  $x_i(t_0)$ . The derivatives of the variables  $x_i$  with respect to  $T$  are:

$$\frac{dx_i}{dT} = -f_i \left[ t_f - T, x_1(t_f - T), x_2(t_f - T), \dots, x_N(t_f - T) \right]$$

as  $t = t_f - T$  and  $dT = -dt$ .

#### E. TEST CASE

To help clarify the writeup, a simple test case is inserted. I am integrating the three equations:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = -4x_1$$

$$\frac{dx_3}{dt} = 2x_3$$

Let  $x_1(0) = 0$ ,  $x_2(0) = 2$ , and  $x_3(0) = 1$ . The solutions of the above differential equations are:

$$x_1 = \sin 2t$$

$$x_2 = 2 \cos 2t$$

$$x_3 = e^{2t}$$

Using the T break feature printing is done every second.  
At  $t$  equal one second.

$$x_1 = \sin 2 = .909297427 \text{ the routine gives } .9092974248$$

$$x_2 = 2 \cos 2 = -.832293674 \text{ the routine gives } .8322936859$$

$$x_3 = e^{2t} = 7.38905610 \text{ the routine gives } 7.389056142$$

The last printout is  $t = 5$  seconds.

$$x_1 = \sin 10 = -.544021111 \text{ the routine gives } -.544021143$$

$$x_2 = 2 \cos 10 = -1.678143058 \text{ the routine gives } 1.678143027$$

$$x_3 = e^{10} = 22026.4658 \text{ the routine gives } 22026.46649$$

## TEST CASE CODING

	ORG	10000	
OUTPUT	EQU	70412B	
ADAMS	EQU	3720B	
TEST	LDA	ONE	
	STA	T	FIRST T BREAK IS 1.
	STA	DATA+ 10	X3 (0) IS 1.
	LDA	TWO	
	STA	DATA+ 9	X2(0) IS 2.
	ENA	0	
	STA	DATA+ 7	+ 0 IS ZERO
	STA	DATA+ 8	X1(0) IS 1.
BETA	SLJ	4 ADAMS	CALLING SEQUENCE
	ZRO	1 DERIV	B NOT ZERO SO PRINT
	ZRO	TEXIT	INITIAL CONDITIONS
	ZRO	T	
+	ZRO	DATA	
	ZRO	COMMON	
+	ZRO	EXIT	
	ZRO	TMIN	
A2	SLS	A2	ERROR RETURN
ONE	DEC	1.	1.
TWO	DEC	2.	2.
SIX	DEC	6.	7.
M FOUR	DEC	-4.	-4.
DATA	DEC	3	CODE IS 3
	DEC	1.	A IN DATA+ 1 IS PLUS
	DEC	1.D-8	E IN DATA+ 2 IS .000001
	DEC	3.D-4	MIN DT IN DATA+ 3 IS .0003
	DEC	1.	MAX DT IN DATA+ 4 IS 1.

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	DEC	3	N IS 3 IN DATA+5
	DEC	5, D-3	INITIAL DT IS .005
	DEC	0, 0, 0, 0, 0, 0, 0	+ X1, X2, X3, and DERIV X1, X2, X3
	DEC	.1, .1, 1.	A1, A2, A3
COMMON	BSS	47	14 W+5 IS 47
T	BSS	1	T BREAK
TMIN	SLS	TMIN	STOP IF DT TOO SMALL
COUNT	BSS	1	
DERIV	SLJ	0	
	LDA	DATA+9	
	STA	DATA+11	DERIV X1 IS X2
	LDA	M FOUR	
	FMU	DATA+8	
	STA	DATA+12	DERIV X2 IS -4 X 1
	LDA	DATA+10	
	FAD	DATA+10	
	STA	DATA+13	2X3 IS DERIV X3
	SLJ	DERIV	
EXIT	SLJ	0	EXIT EACH DT STEP
	SLJ	EXIT	
TEXIT	SLJ	0	
	SLJ	4 OUTPUT	PRINT TITLE
A3	SLS	A3	
+	03	BCD	SUBSCRIPT
	00	2021	
+	03	BCD+2	
	00	1029	X
+	03	BCD+3	
	00	1047	DERIV X
+	03	BCD+4	
	00	1070	T
+	01	4 4	
	00	2 10	PRINT TITLE



	ENA	1	
	STA	COUNT	
	ENI	1 0	
LOOP	SLJ	4	OUTPUT
A4	SLS	A4	
+	04	COUNT	COUNT 1 TO 3
	00	10	
+	06	1	DATA+ 8
	00	10030	X1
+	06	1	DATA+ 11
	00	10050	DERIV X1
+	06		DATA+ 7
	00	10070	TIME
+	01	4 4	
	00	2 16	PRINT X. DERIV X. AND T
	RAO	COUNT	EVERY SEC
+	ISK	1 2	BUMP 1
	SLJ	LOOP	
	LDA	T	END PRINT
	FAD	ONE	
	STA	T	
	FSB	SIX	
	AJP	M	TEXT
STOP	SLS	STOP	IF DATA+7 OR T IS 7 STOP
BCD	BCD	2SUBSCRIPT	
	BCD	1X	
	BCD	1	DERIV X
	BCD	1	T
	END	TEST	

## TEST CASE OUTPUT

CODE	A	E	MIN DT	MAX DT	N	DELTA T
3	.10 +1	.10 -9	.30 -3	.10 +1	3	.5000000000 -2
SUBSCRIPT	X		DERIV X	A	T	
1		0	.2000000000 +1	.1000000000 +0		0
2	.2000000000 +1	+1	0	.1000000000 +0		0
3	.1000000000 +1	+1	.2000000000 +1	.1000000000 +1		0

SUBSCRIPT	X		DERIV X	T		
1	.9092974248 +0	+0	-.8322924574 +0	.1000000000 +1		
2	-.8322936859 +0	+0	-.3637190736 +1	.1000000000 +1		
3	.7389056142 +1	+1	.1477812185 +2	.1000000000 +1		

SUBSCRIPT	X		DERIV X	T		
1	-.7568025058 +0	+0	-.1307288205 +1	.2000000000 +1		
2	-.1307287223 +1	+1	.3027208220 +1	.2000000000 +1		
3	.5459815071 +2	+2	.1091963721 +3	.2000000000 +1		

SUBSCRIPT	X		DERIV X	T		
1	-.2794154782 +0	+0	.1920340542 +1	.3000000000 +1		
2	.1920340590 +1	+1	.1117662231 +1	.3000000000 +1		
3	.4034288012 +3	+3	.8068576686 +3	.3000000000 +1		

SUBSCRIPT	X		DERIV X	T		
1	.9854519960 +0	+0	-.2909987991 +0	.4000000000 +1		
2	-.2910001235 +0	+0	-.3957433293 +1	.4000000000 +1		
3	.2980958060 +4	+4	.5961919977 +4	.4000000000 +1		

SUBSCRIPT	X		DERIV X	T		
1	-.5440211430 +0	+0	-.1678143115 +1	.5000000000 +1		
2	-.1678143027 +1	+1	.2176084290 +1	.5000000000 +1		
3	.2202646649 +5	+5	.4405293659 +5	.5000000000 +1		

f. INTERPF.

A. IDENTIFICATION

TITLE: Divided Difference Interpolation or  
Extrapolation Routine  
CATEGORY: Mathematical Subroutine  
PROGRAMER: Sanford Elkin, William Silverman, and  
Ed Fleming  
MODIFIED: June 1961

B. PURPOSE:

Given a table of M values in floating point which define a function, and given a floating point argument X, this routine approximated the functional value Y of that argument by a polynomial interpolation or extrapolation.

C. USAGE:

1. Calling sequence:

CALL INTERPF (X, M, N, a, b)

Interpolate, find Y for X,	X Flt argument
where various X vs. Y given	M Size of the table
in table.	N Degree of interpolation
	a Table of X values
	b Table of Y values

Where M is in fixed integer form, N is the order of interpolation or extrapolation desired — in fixed integer form.

The X's and Y's must be in floating format and the table of X values must be stored in decreasing order.

## D. MATHEMATICAL METHOD: (reference 19)

$$Y = I_{0,1,\dots,n}(x) = \frac{1}{x_n - x_0} \begin{vmatrix} I_{0,1,\dots,n-1}(x) & x_0 - x \\ I_{1,2,\dots,n}(x) & x_n - x \end{vmatrix}$$

$Y = I_{0,1,\dots,n}(x)$  is equivalent to Lagrange's Formula

$$\begin{aligned} Y = \psi(x) = & \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} Y_0 \\ & + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} Y_1 \\ & + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} Y_n \end{aligned}$$

which yield a polynomial of degree  $n$  such that  $\psi(x_0) = Y_0$ ,  $\psi(x_1) = Y_1$ ,  $\dots$ ,  $\psi(x_n) = Y_n$ , as is easily demonstrated by an induction proof.

When  $x$  is within the limits of the table, the points  $x_0, x_1, \dots, x_n$  are spaced about it to best advantage, but if  $x$  is outside the limits of the table, the nearest  $n+1$  points in the table are used in the formula.

g. RDF.

**TITLE:** Read Data, Fixed Formats  
**ID:** RDF  
**Classification:** Input Routine  
**Programers:** W. Silverman and R. E. Mann

**PURPOSE:**

To load data from Hollerith cards through the 088 card reader, or from 80 character records on BCD tape. The data are on symbolic cards of the types used for the pseudo operations DEC, OCT, and BCD as described in the assembly routine.

**USAGE:**

## 1. Calling Sequence

	ENA	A
	ENQ	B
$\alpha$	RTJ	RDF
$\alpha + 1$	NOP	L(LTN)
$\alpha + 2$	Error	Return
$\alpha + 3$	Normal	Return

L(LTN) = Location of Logical Tape Number

LTN (Logical Tape Number) = 1-48 for tape input  
                                   = 49 for card input

After a description of the input formats, the function of A and B will be discussed.

## 2. Input Formats:

Load is controlled by a symbolic operation code in columns 10, 11, 12 of the input cards (or records). There are five permissible operation codes.

1. SLJ--This operation always causes loading to be terminated. A transfer address may appear in decimal or octal

beginning in column 20. The address is assumed decimal if it is followed by a D or blank, and octal if it is followed by a B. If index modification of the transfer address is desired, column 17 or 18 may contain an index register number. Control is transferred to the (modified) transfer address, or if there is no transfer address, control is returned to  $a + 3$  in the calling sequence.

2. REM-- This record will be skipped. No conversion or operation results. REM cards may be used as spacers or tags in the data deck.

Conversion operations:

3. BCD-- $n$  words of binary coded decimal information beginning in column 21 are loaded into consecutive memory locations.  $n$  is in column 20,  $1 \leq n \leq 7$ . The information runs through column  $20 + 8n$ .

4. OCT--Octal data beginning in column 20 and terminating with the first blank column are interpreted as octal integers, and converted to binary integers. Successive words are separated by commas and loaded into consecutive locations. Each word may consist of a + or - sign and up to 16 octal digits. If no sign appears, a + sign is assumed.

5. DEC--Decimal data beginning in column 20 and terminating with the first blank column are loaded. Successive words are separated by commas and loaded into consecutive locations. Each word may consist of a sign, + or - or none, up to 15 decimal digits with a decimal point if desired, a D or E followed by a signed or unsigned decimal scale factor, and a B followed by a signed or unsigned binary scale factor. Presence of a binary scale factor will cause the number to be loaded as a fixed point decimal number with the binary point to the right of the bit position given by the binary scaling. If a decimal scale factor or decimal point is present, the number will be loaded as a fixed point number as above if

a binary scaling is present, or as a (scaled) floating point number if no binary scaling is present. If a binary scaling is present, it must lie between -47 and 47, and the decimal scaling, if any, must lie between -28 and 28. If no decimal point or scale factor is present, the number will be loaded as an integer. A few illustrative examples follow:

- 1) 1.2345 will be loaded as floating point 1.2345
- 2) 12345E-3 will be loaded as floating point 12.345
- 3) 1.2345D-2 will be loaded as floating point .012345
- 4) 12345 will be loaded as integer 12345
- 5) 12345B15 will be loaded as fixed point 12345.0 with binary point to the right of bit position 15.
- 6) 12.345D2B13 will be loaded as fixed point 1234.5 with binary point to the right of bit position 13.
- 7) 12345B-3 will be loaded as fixed point 12345.0 with binary point to the right of bit position -3, that is as (rounded) integer  $1543 = 12345/8$  to the nearest integer.

Any data card BCD, OCT, or DEC may have an absolute numerical address in columns 1-8. The data on the card is loaded relative to that address. The address is treated as octal or decimal according to the same conventions used for the transfer address on the SLJ card.

e. g. The card

2000B      BCD    2ABCDEFGH IJKLMN OP

will be loaded so that:

(2000B) =    ABCDEFGH

(2001B) =    IJKLMN OP

A and B (the addresses in the accumulator and quotient registers upon entry to RDF) condition where the data on cards with blank address fields is loaded and the number of words actually loaded by RDF.

If  $A \neq 0$ , the first card with a blank address field is loaded relative to A, the data going into addresses A, A+1, ...,

$A+n-1$ , where  $n$  is the number of data words on the card. The data from the next card with a blank address field is loaded relative to  $A$  into locations  $A+n$ ,  $A+n+1, \dots, A+n+m-1$  where  $m$  is the number of data words on the card. And so on, each card with a blank address field being loaded relative to  $A$ .

If  $A = 0$ , the first card must have an absolute numerical address in columns 1-8. Loading proceeds relative to the last such address. Thus if the first data card has the address 2000B, the second card has a blank address field, the third has the address 3000B, and the remaining data cards have blank address fields, the data from the first cards will go into consecutive addresses starting at 3000B. If  $B \neq 0$ ,  $B-A+1$  words will be loaded unless the load is terminated by an SLJ card or an error (see errors below).

If  $B = 0$ , an indefinite number of words will be loaded until the routine is terminated by an SLJ card or an error.

3. RDF requires 397 locations
4. RDF uses 19 common erasable storage locations
5. Errors:

1. End of file on tape or card.

The end of file flag (77713B) is set non-zero and if no other errors occurred, control is returned to  $a + 3$  in the calling sequence.

2. Parity error on tape.

The RTT flag (77712B) is set non-zero, and the routine continues, although the conversion of the record on which the parity error occurred may be inaccurate. When the routine is terminated, control is returned to the error return,  $a + 2$ , in the calling sequence, with the number of records which had parity errors in the upper address of the  $A$  register, and the transfer address, if



any, in the Q register.

### 3. Illegal punch on card.

The RTT flag is set with 1's in bits 0 through 39 corresponding to erroneous columns on the card (these may apply to the first or second half of the card, but not both). Control is returned to  $a + 2$  in the calling sequence, with the upper address of A set to 1.

### 4. Format error:

This may be caused by an illegal operation code in columns 10, 11, 12; an illegal address; an illegal character in column 17 or 18 of an SLJ card; or an illegal character in column 20 of a BCD card. Control is restored to  $a + 2$  in the calling sequence with bit 47 of A set to 1.

### 5. Data error:

This may be caused by an illegal character in a numerical field; more than 16 digits in an octal numerical field, or more than 14 in a decimal numerical field; more than one sign in a numerical field or a sign which is preceded by a number in the field; more than one decimal point in a decimal numerical field; too large a scale factor; a decimal number which when scaled does not fit the A register, or which is too large or too small for the floating point format

$$(\geq -2^{1023} \text{ or } \leq -2^{-1023}).$$

In case of a data error, the word in storage corresponding to the erroneous field is set equal to minus zero, and loading continues. At the end of the routine, control is returned to  $a + 2$  in the calling sequence, with the number of erroneous fields in the lower address of the A register.

### 6. Termination of loading:

An SLJ card, an end-of-file, or a format error automatically terminates loading. If  $B \neq 0$  (in the Q register

of entry), loading is terminated when  $B-A+1$  words have been loaded. The program then hunts forward through the remaining cards (or records on tape) to the first SLJ card, or end-of-file. The tape or card reader is left set to read the next succeeding record or card. Normal return in this case is to  $a + 3$  in the calling sequence.

7. All conversions are performed rapidly enough so that the 1607 tape mechanism (or the 088 card reader) can be operated at full speed; 3000 records per minute on tape or 650 cards per minute through the 088.

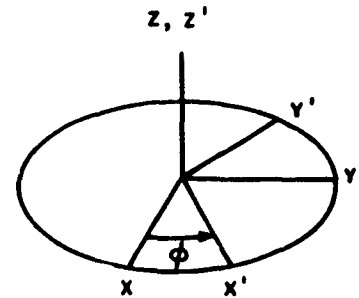
8. Floating point numbers with large negative exponents may be accurate to only 35 bits.

h. Rotate.

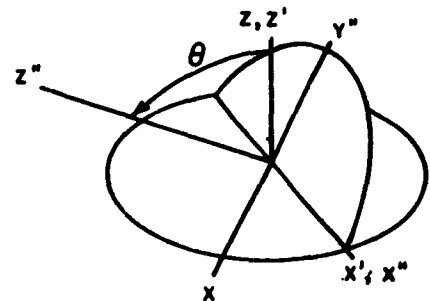
## MATRIX ROTATION

The following transformation was used as a subroutine to transform from one Cartesian coordinate system to another Cartesian coordinate system.

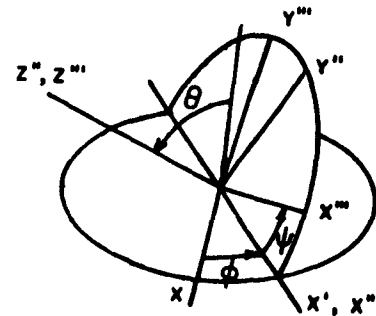
1. Rotation of the initial system by an angle  $\phi$  counterclockwise about the Z axis.



2. Rotation of the primed system by the angle  $\theta$  counterclockwise about the X' axis.



3. Rotation of the double primed system by the angle  $\psi$  counterclockwise about the Z'' axis.



$$\begin{vmatrix} X''' \\ Y''' \\ Z''' \end{vmatrix} = \begin{vmatrix} A \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

Figure 6.

$$A = \begin{vmatrix} \cos \psi \cos \phi - \cos \theta \sin \phi \sin \psi & -\sin \psi \cos \phi - \cos \theta \sin \phi \cos \psi & \sin \theta \sin \phi \\ \cos \psi \sin \phi + \cos \theta \cos \phi \sin \psi & -\sin \psi \sin \phi + \cos \theta \cos \phi \cos \psi & -\sin \theta \cos \phi \\ \sin \theta \sin \psi & \sin \theta \cos \psi & \cos \theta \end{vmatrix}$$

This program is callable by FORTRAN.

To use:

CALL ROTATE (A, B, C, D, E)

where

A = The angle  $\varphi$

B = The angle  $\theta$

C = The angle  $\psi$

D = The input 3 component vector

E = The output 3 component vector

i. SETTAB

The Subroutine SETTAB is used to set up a table of print times. This table includes both even increment print times and abnormal print times such as ignition, drop stages, and burnout.

The subroutine is callable by FORTRAN. To use:

CALL SETTAB (NS, TMI, TMCC, TMBO, DT, TABLE, IX)

where

NS	=	number of stages
TMI	=	table of ignition times
TMCC	=	table of coefficient change times
TMBO	=	table of burnout time
DT	=	even time increments
TABLE	=	resulting table
IX	=	size of the table

j. TWO BOD

## SYMBOLS

ENGLISH

a	-	Semi-major axis of orbit	(NM)
$a_E$	-	Equatorial radius of the earth	(NM)
b	-	Semi-minor axis of orbit	(NM)
C	-	Earth parameter defined by equation 3-1	(NM)
e	-	Eccentricity of the orbit	
$e_E$	-	Eccentricity of the earth ( $e_E^2 = 2f - f^2$ )	
E	-	Eccentric anomaly	(RAD)
f	-	Flattening of the earth	
GM	-	Gravitational constant of the earth	(FT <sup>3</sup> /SEC <sup>2</sup> )
$h_G$	-	Geodetic altitude	(NM)
$h_x, h_y, h_z$	-	Angular momentum about subscripted axis	
h	-	Total angular momentum	
$h_A$	-	Apogee altitude	(NM)
$h_p$	-	Perigee altitude	(NM)
H	-	Energy parameter defined by equation 2-2	
i	-	Inclination of orbit plane to equatorial plane	(DEG)
k	-	Constant used to obtain period ( $2\pi/\sqrt{GM}$ )	
M	-	Mean anomaly	
P	-	Period of the orbit	(MIN)
R	-	Geocentric radius from center of earth to vehicle	(NM)
$\dot{R}$	-	Change in R with respect to time	(NM/SEC)
R'	-	Geocentric radius used for earth's velocity	(NM)
S	-	Earth parameter defined by equation 3-2	(NM)
$t_o$	-	Time of entry into orbit	(SEC)

## SYMBOLS

$T$	- Time of perigee passage	(SEC)
$t_i$	- Time at $i^{\text{th}}$ point in orbit	(SEC)
$V_E$	- Velocity of vehicle with respect to the earth	(FPS)
$V_R$	- Velocity of earth's surface below vehicle	(FPS)
$V_I$	- Velocity of vehicle with respect to non-rotating earth	(FPS)
$x_b, y_b, z_b$	- Earth centered coordinates of the space vehicle	(Figure 3)
$x, y, z$	- Geodetic coordinate system at vehicle	(Figure 4)
$x', y', z'$	- Geodetic coordinate system below vehicle on surface of the earth	(Figure 4)
$x'', y'', z''$	- Coordinate system in orbit plane	(Figure 2)
$X, Y, Z$	- Earth centered coordinate system	(Figure 1)
$x_s, y_s, z_s$	- Coordinates of the look station	(Figure 3)

GREEK

$\alpha$	- Angle between geodetic east axis and X axis of coordinate system tangent to a geodetic earth; also the aspect angle	(DEG)
$\beta$	- Initial azimuth (positive c. w. from north)	(DEG)
$\gamma$	- Initial flight path angle (positive up)	(DEG)
$\gamma_I$	- Inertial flight path angle (positive up)	(DEG)
$\omega$	- Argument of perigee	(DEG)
$\omega_E$	- Rotational rate of the earth	(RAD/SEC)
$\Omega$	- Longitude of ascending node	(DEG)
$\varphi$	- Longitude	(DEG)
$\theta_G$	- Geodetic latitude	(DEG)
$\theta_c$	- Geocentric latitude from vehicle	(DEG)
$\theta'_c$	- Geocentric latitude from earth's surface	(DEG)
$\mu$	- Argument of latitude	(DEG)
$v$	- True anomaly	(DEG)

SUBSCRIPTS

- b - refers to space vehicle
- c - refers to geocentric
- E - refers to the earth
- G - refers to geodetic
- i - refers to  $i^{\text{th}}$  point in orbit
- I - refers to nonrotating earth
- o - refers to initial or burnout values
- R - refers to rotational velocity radius
- s - refers to look station
- x, y, z - refers to X, Y, Z, coordinate system
- 1 - refers to coordinate system at station parallel to equatorial plane
- 2 - refers to coordinate system at look station

A dot over a variable indicates the time derivative of that variable.

A delta ( $\Delta$ ) in front of a variable indicates a difference in that variable.



# 1. INTRODUCTION.

## a. Inputs.

The inputs are a position and velocity vector in a right-hand Earth-centered coordinate system with the X axis along the node of the equatorial plane and the Greenwich meridian and the Z axis along the north polar axis. The number of time increments, changes, and look-angle stations must be in COMMON. To use the subroutine, control number 4 must be set equal to 1 or 2.

## b. Printout time changes.

Since some portions of the trajectory are more important than others, provisions are included for changing the time increment, with the use of a maximum of five different time increments. If the ellipse intersects the earth, the trajectory computation stops at this time. If it does not intersect the earth, the computation will continue for a prespecified number of orbits.

## c. TWO-BOD computer program.

The program computes the classical orbital elements (a, b, e, P,  $h_A$ ,  $h_P$ ,  $i$ ,  $\omega$ ,  $\Omega$ ,  $v_0$ ,  $E_0$ ).

After the orbital elements are computed, a matrix-rotation is set up to transfer from the orbital plane to the equatorial plane coordinate system. Time is incremented and a new true anomaly is obtained.

The corresponding orbit plane coordinates are then transformed by a matrix rotation to the equatorial coordinate system. The latitude and longitude are obtained by taking "arc tangents" and adding the effects of a rotating earth. These values are stored along with the radius vector to the vehicle and are used to find the "look angles" at various stations.

The matrix subroutine described in rotate is used for the coordinate transformation.

2. EQUATIONS OF CELESTIAL MECHANICS.a. Semi-major axis.

$$V_I = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2} \quad (1)$$

$$H = \frac{RV_I^2}{GM}, \text{ if } H \geq 2, \text{ the rocket escapes} \quad (2)$$

$$a = \frac{R}{2-H} \quad (3)$$

b. Angular momentum.

$$h_x = Y\dot{Z} - Z\dot{Y} \quad (4a)$$

$$h_y = Z\dot{X} - X\dot{Z} \quad (4b)$$

$$h_z = X\dot{Y} - Y\dot{X} \quad (4c)$$

$$h = \sqrt{h_x^2 + h_y^2 + h_z^2} \quad (4d)$$

c. Orbital elements.

$$i = \cos^{-1} (h_z/h) \quad (5)$$

$$\Omega = \tan^{-1} \left( \frac{h_x}{-h_y} \right) \quad \text{If } (-h_y) \text{ is negative,} \quad (6)$$

$$\Omega = \Omega + 180^\circ$$

$$R = \frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{R} \quad (7)$$

$$e = \left( 1 - \frac{h^2}{GMa} \right)^{1/2} \quad (8)$$

$$\cos v_o = \frac{h^2}{RGM} - 1 \quad (9)$$

$$\sin v_o = \frac{h\dot{R}}{GM} \quad (10)$$

$$v_o = \tan^{-1} \left( \frac{\sin v_o}{\cos v_o} \right) \quad \text{If } (\cos v_o) \text{ is negative,} \quad (11)$$

$$v_o = v_o + 180^\circ$$

$$\cos \mu = \frac{(Y h_x - X h_y)}{h} \quad (12)$$

$$\mu = \tan^{-1} \left( \frac{Z}{\cos \mu} \right) \quad \text{If } (\cos \mu) \text{ is negative} \quad (13)$$

$$\mu = \mu + 180^\circ$$

$$\omega = \mu - v_o \quad (14)$$

$$b = a \sqrt{1 - e^2} \quad (15)$$

$$P = k a^{3/2} \quad (16)$$

$$h_A = a(1 + e) - a_E \quad (17)$$

$$h_P = a(1 - e) - a_E \quad (18)$$

$$E_o = 2 \tan^{-1} \left( \sqrt{\frac{1 - e}{1 + e}} \tan \frac{v_o}{2} \right) \quad (19)$$

$$t_o - T = \frac{(E_o - e \sin E_o) P}{2\pi} \quad (20)$$

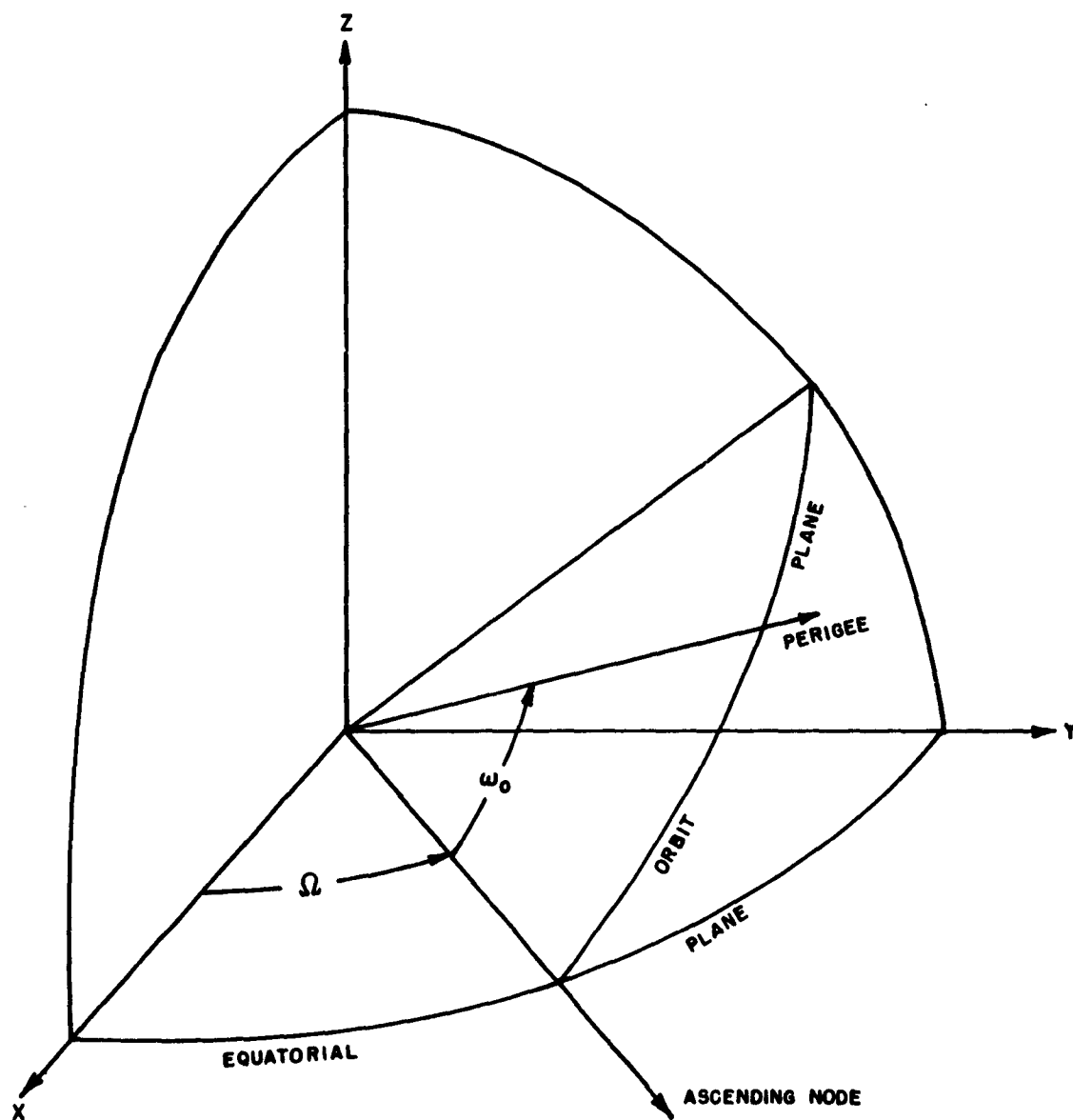


Figure 1.

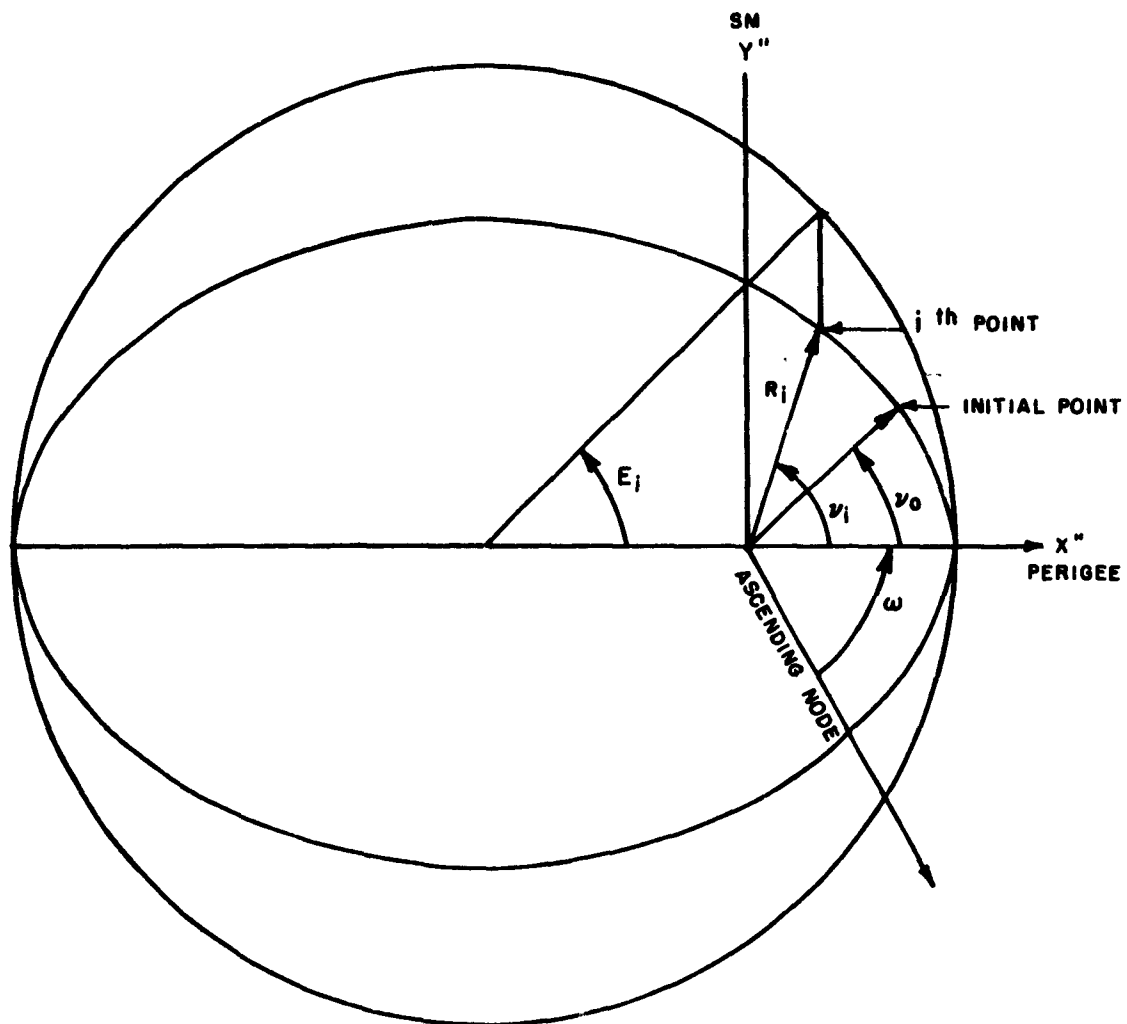


Figure 2

d. Trajectory points.

$$t_{i+1} = t_i + \Delta t \quad (21)$$

$$M = \frac{2\pi}{P} (t_i - T) \quad (22)$$

$$E_2 = E_1 - \frac{(E_1 - e \sin E_1 - M)}{1 - e \cos E_1} \quad \begin{array}{l} \text{Iterate until} \\ E_2 = E_1 \end{array} \quad (23)$$

$$v_i = 2 \tan^{-1} \left( \sqrt{\frac{1+e}{1-e}} \tan \frac{E_i}{2} \right) \quad (24)$$

$$R_i = \frac{a(1-e^2)}{1+e \cos v_i} \quad (25)$$

$$V_i = \left[ GM \left( \frac{2}{R_i} - \frac{1}{a} \right) \right]^{1/2} \quad (26)$$

$$\gamma_i = \tan^{-1} \left[ \frac{e \sin v_i}{1 + e \cos v_i} \right] \quad (27)$$

$$x'' = \cos v_i \quad (28a)$$

$$y'' = \sin v_i \quad (28b)$$

$$z'' = 0 \quad (28c)$$

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{vmatrix} \begin{vmatrix} \cos \omega & -\sin \omega & 0 \\ \sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} \quad (29)$$

$$\begin{aligned} X &= x'' (\cos \Omega \cos \omega - \sin \Omega \cos i \sin \omega) - y'' (\cos \Omega \sin \omega + \sin \Omega \cos i \cos \omega) \\ Y &= x'' (\sin \Omega \cos \omega + \cos \Omega \cos i \sin \omega) - y'' (\sin \Omega \sin \omega - \cos \Omega \cos i \cos \omega) \\ Z &= x'' (\sin i \sin \omega) + y'' (\sin i \cos \omega) \end{aligned} \quad (30)$$

$$\theta_c = \tan^{-1} \frac{Z}{\sqrt{X^2 + Y^2}} \quad (31)$$

$$\varphi_c = \tan^{-1} \left( \frac{Y}{X} \right) \quad \text{If } X \text{ is negative, } \varphi_c = \varphi_c + 180^\circ \quad (32)$$

$$\varphi_c = \varphi_c - \omega_E (t_1 - t_0) \quad (33)$$

e. Geocentric to geodetic.

$$\theta_G = \theta_c + \sin^{-1} \left\{ \frac{a_E}{R_1} \left[ f \sin 2\theta_c + f^2 \sin 4\theta_c \left( \frac{a_E}{R_1} - \frac{1}{4} \right) \right] \right\} \quad (34)$$

$$h_G = R_1 - a_E \left[ 1 - f \sin^2 \theta_c - \frac{f^2}{2} \sin^2 2\theta_c \left( \frac{a_E}{R_1} - \frac{1}{4} \right) \right] \quad (35)$$

f. Range.

The great circle range is obtained by computing the range angle. This angle is obtained by taking the dot product of the launch vector with the radius vector at any given time.

$$\begin{aligned} \text{R.A.} &= \cos^{-1} \left\{ (X_L X + Y_L Y + Z_L Z) / \sqrt{X_L^2 + Y_L^2 + Z_L^2} \sqrt{X^2 + Y^2 + Z^2} \right\} \\ \text{Range} &= R_E (\text{avg}) (\text{R.A.}) \end{aligned} \quad (36)$$

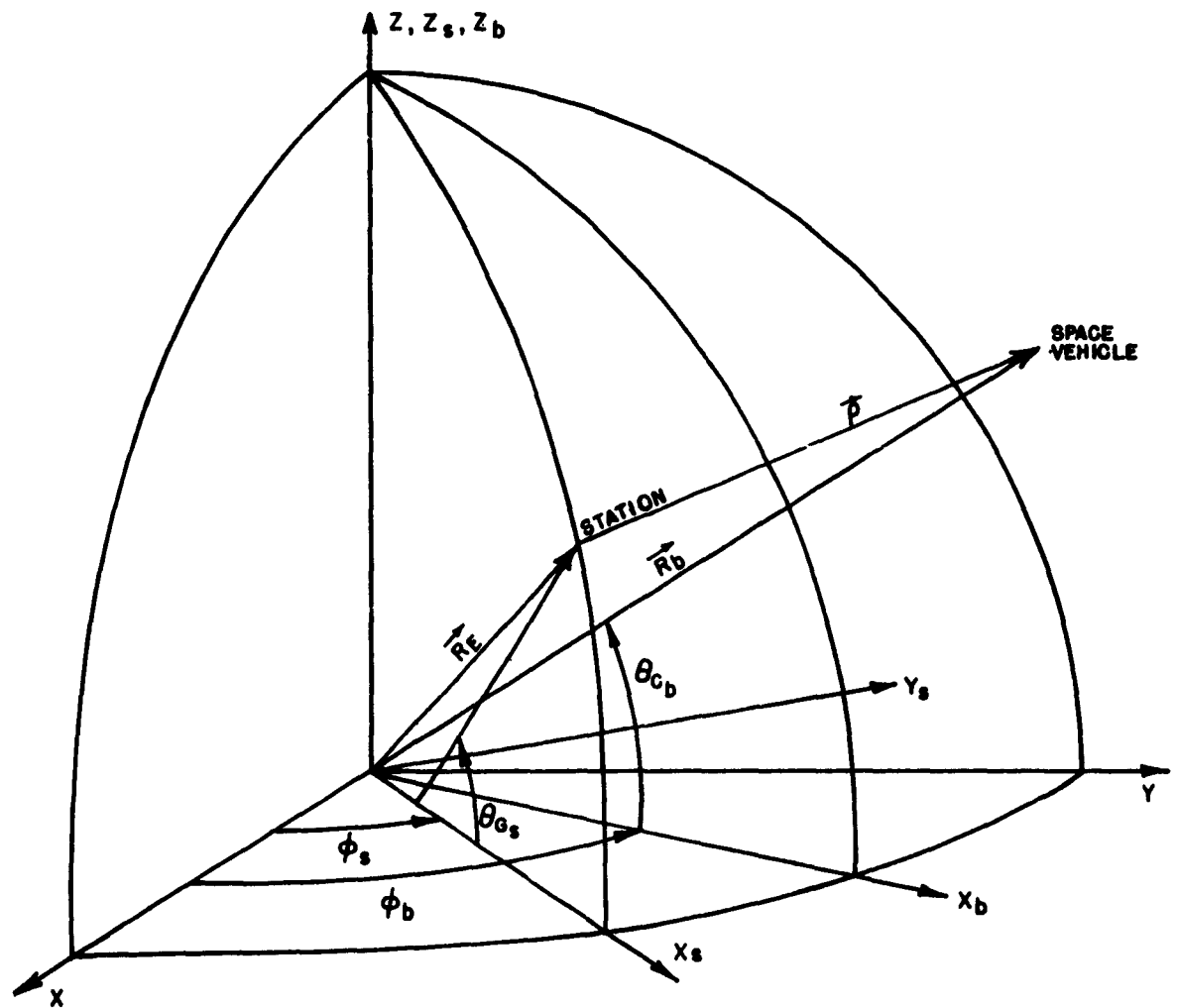


Figure 3



g. Look angles.Station given station  $(h_{G_s}, \theta_{G_s}, \varphi_s)$ 

$$C_s = \frac{a_E}{(1 - e_E^2 \sin^2 \theta_{G_s})^{1/2}} \quad S_s = C_s (1 - e_E^2) \quad (37)$$

$$|\vec{R}_E| = \left[ (S_s + h_{g_s})^2 \sin^2 \theta_{G_s} + (C_s + h_{g_s})^2 \cos^2 \theta_{G_s} \right]^{1/2} \quad (38)$$

$$\theta_{c_s} = \tan^{-1} \left[ \left( \frac{S_s + h_{g_s}}{C_s + h_{g_s}} \right) \tan \theta_{G_s} \right] \quad (39)$$

$$R_E \begin{cases} x_s = R_E \cos \theta_{c_s} \\ y_s = 0 \\ z_s = R_E \sin \theta_{c_s} \end{cases} \quad (40)$$

$$R_b \begin{cases} x_b = R_b \cos \theta_{c_b} \cos \Delta \varphi \\ y_b = R_b \cos \theta_{c_b} \sin \Delta \varphi \\ z_b = R_b \sin \theta_{c_b} \end{cases} \quad \text{where } \Delta \varphi = \varphi_b - \varphi_s \quad (41)$$

$$\vec{R}_b = \vec{R}_E + \vec{\rho}$$

$$\vec{\rho} = \vec{R}_b - \vec{R}_E \quad (42)$$

$$\begin{cases} x_1 = R_b \cos \theta_{c_b} \cos \Delta \varphi - R_E \cos \theta_{c_s} \\ y_1 = R_b \cos \theta_{c_b} \sin \Delta \varphi \\ z_1 = R_b \sin \theta_{c_b} - R_E \sin \theta_{c_s} \end{cases} \quad (43)$$

$$\begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} = \begin{vmatrix} \cos \theta_G & 0 & -\sin \theta_G \\ 0 & 1 & 0 \\ \sin \theta_G & 0 & \cos \theta_G \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} \quad (44)$$

$$\text{Elevation ang.} = \tan^{-1} \left[ \frac{z_2}{\sqrt{x_2^2 + y_2^2}} \right] \quad (45)$$

$$\text{Azimuth ang.} = \tan^{-1} \left[ y_2/x_2 \right] \quad \begin{array}{l} \text{If } x_2 \text{ neg,} \\ \text{add } 180^\circ \end{array} \quad (46)$$

If (azimuth) is neg, add  $360^\circ$

#### h. Aspect angle.

The aspect angle is defined as the angle between the spin axis of the space vehicle and the vector from the look station to the vehicle. This program assumes the vehicle remains fixed in space. The angle is obtained by the following equations:

$$a = \cos^{-1} \left[ (\dot{X}x_1 + \dot{Y}y_1 + \dot{Z}z_1) / \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2} / \sqrt{x_1^2 + y_1^2 + z_1^2} \right] \quad (47)$$

## VARIABLES USED IN TWO BODY SUBROUTINE

ALT	Geocentric radius to vehicle
AMU	Argument of latitude
ANS	Output vector
ANT	Aspect angle
APOGEE	Apogee altitude of the orbit
ARA	Average radius of the Earth
AX	X Component of inertial burnout position vector
AXDOT	X Component of inertial burnout velocity vector
AY	Y Component of inertial burnout position vector
AYDOT	Y Component of inertial burnout velocity vector
AZ	Z Component of inertial burnout position vector
AZDOT	Z Component of inertial burnout velocity vector
AZIM	Azimuth of look vector
A1	Burnout position vector
A2	Burnout velocity vector
A1A	Not used in two body *
A1B	Not used in two body *
C	$\Delta = R_{ee} / \sqrt{1 - e^2 \sin^2 \theta_c}$
CAPO	Longitude of ascending node
CLAL	Cosine of the launch latitude *
CLOL	Cosine of the launch longitude
CMU	Cosine of MU
CSLANT	Cosine of inclination angle
CT	Cosine of geocentric latitude
CTRVA	Cosine of true anomaly
DELTAT	Stored time increment *
DPHI	Difference in longitude
DUMTB	Not used in two body *
E	Eccentricity of the Earth

---

\* Stored in COMMON

ELEV	Elevation of look angle vector
ENDT	End of time increment *
ER	Size of allowable error
ETA	Anomaly = period/ $2\pi$
EX	Orbital eccentricity
EXANOM	Eccentric anomaly
EXIMP	Impact eccentric anomaly
E1	Used to compute eccentric anomaly
E2	Used to compute eccentric anomaly
F	Flattening of the Earth
FMIN	Output - time in minutes
FMINF	Function for obtaining FMIN
FPA	Flight path angle
FX	Output vector from Rotate
GM	Earth gravitational attraction constant
GM1	Earth gravitational attraction constant
H	Angular momentum parameter
HH	Angular momentum squared
HOURL	Output time in hours
HOURLH	Function for obtaining HOURL
HX	X Component of angular momentum
HY	Y Component of angular momentum
HZ	Z Component of angular momentum
H1	Angular momentum
I	Utility index
IT	Utility index
J	Utility index
K	Utility index
KBATT	Number of Batt output tape *
MEX	Utility index
MM	Utility index
NAME	Name stored in common *

---

\* Stored in COMMON

NAT	Number of output tape *
NOST	Number of look angle stations *
NOT	Number of time changes *
N1C	Not used in two body *
ORBT	Number of orbits of orbital vehicle *
PERIGEE	Perigee altitude of the orbit
PERIOD	Time to make one revolution of orbit
PHI	Stored longitude
PHIX	Used to compute longitude
PI	$= \pi = 3.141592654$
PN	Page count
POP	Earth flattening constant
PRALT	Geodetic latitude of the vehicle
PRLAT	Geodetic altitude of the vehicle
PRLON	Longitude of the vehicle
PRTIME	Time from launch
R	Length of position vector n. m.
RAD	$= \pi / 180. = 1/57.2957795$
RANGE	Great circle range on surface of Earth
RDOT	Rate of change along R with respect to time
RE	Radius of Earth = $f(\theta_c)$
REE	Equatorial radius of Earth - n. m.
RF	Length of position vector (ft)
RLAU	Radius to launch point *
RS	Radius of look station
S	$\frac{\Delta C}{\Delta} (1 - e^2)$
SEC	Output time in seconds
SECF	Function for obtaining output time in seconds
SHT	Look station altitude
SLAL	Sine of the launch latitude *
SLANT	Orbital inclination angle
SLOL	Sine of the launch longitude
SLR	Distance from tracking station to vehicle

---

\* Stored in COMMON

<b>SMAJ</b>	<b>Semi major axis</b>
<b>SMIN</b>	<b>Semi minor axis</b>
<b>SMON</b>	<b>Argument of perigee</b>
<b>SPH</b>	<b>Look station longitude</b>
<b>SSLANT</b>	<b>Sine of inclination angle</b>
<b>ST</b>	<b>Sine of theta</b>
<b>STH</b>	<b>Look station latitude</b>
<b>STHC</b>	<b>Look station geocentric latitude</b>
<b>STRVA</b>	<b>Sine of the true anomaly</b>
<b>SX</b>	<b>Orbital plane coordinate system vector</b>
<b>S1</b>	<b>Variables used to compute output variables</b>
<b>S2</b>	<b>Variables used to compute output variables</b>
<b>S3</b>	<b>Variables used to compute output variables</b>
<b>S4</b>	<b>Variables used to compute output variables</b>
<b>S5</b>	<b>Variables used to compute output variables</b>
<b>SP6</b>	<b>Used to calculate aspect angle</b>
<b>TAG</b>	<b>Not used in two body *</b>
<b>THETA</b>	<b>Geocentric latitude</b>
<b>TIME</b>	<b>Independent variable</b>
<b>TIMP</b>	<b>Time to impact</b>
<b>TINCR</b>	<b>Time increment</b>
<b>TOPP</b>	<b>Time of perigee passage</b>
<b>TRANOM</b>	<b>Initial true anomaly</b>
<b>TRIMP</b>	<b>Impact point true anomaly</b>
<b>TRUNU</b>	<b>True anomaly</b>
<b>VELOC</b>	<b>Velocity at any point of the orbit</b>
<b>VI</b>	<b>Initial inertial velocity</b>
<b>WE</b>	<b>Rotational velocity of the Earth</b>
<b>X</b>	<b>X Component of look vector</b>
<b>XAZ</b>	<b>Azimuth of look vector</b>
<b>XEL</b>	<b>Elevation of look vector</b>
<b>XIM</b>	<b>X Component of position vector for range computations</b>

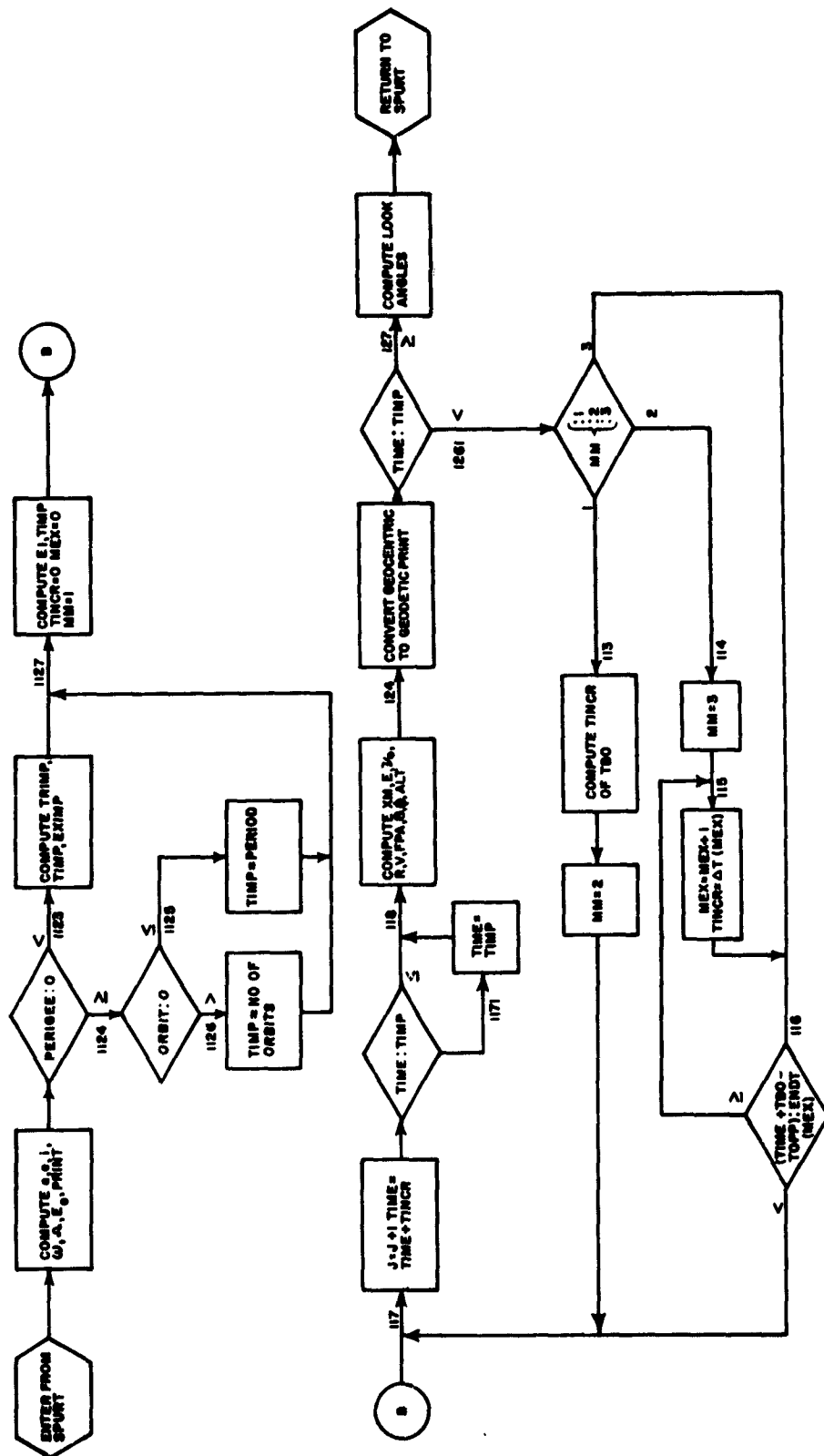
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\* Stored in COMMON

XLAV	X Component of launch vector
XLLO	Launch longitude *
XM	Mean anomaly
XP	Used in matrix rotation
XSHT	Stored tracking station altitude *
XSPH	Stored tracking station longitude *
XSTH	Stored tracking station latitude *
XX	$= \sqrt{\frac{1-e}{1+e}}$ , storage variable
XX1	Sine of station geodetic latitude
XX2	Cos of Station geodetic Latitude
XX3	Used in station radius calculation
XX4	Used in station radius calculation
Y	Y Component of look vector
YIM	Y Component of position vector for range computation
YLAU	Y Component of launch vector
YY	Storage variable
Z	Z Component of look vector
ZIM	Z Component of position vector for range computation
ZIZ	Range angle
ZLAU	Z Component of launch vector
ZP	Used in matrix rotation
Z2	Number of lines per page *

---

\* Stored in COMMON





k. WOTF. SAUF.

A. IDENTIFICATION

TITLE: WRITE ON TAPE  
CO-OP ID: WOTF  
CATEGORY: General Tape Handler  
Programmer: J. W. Wise  
Date: August 4, 1961

B. PURPOSE:

This routine is used to write a record of arbitrary length on a magnetic tape in either binary or BCD mode

C. USAGE:

1. Calling Sequence

CALL WOTF (a, b, n, m)

n = 1-48, MT  
m = 0, BINARY  
m = 1, BCD  
a = First LOCN  
b = Last LOCN

CLASSIFICATION: Utility Routine

TITLE: SAVE TAPE

ID: SAVF

PURPOSE: To inform the operator that a tape is to be saved and to rewind the tape with interlock. E. O. F. and 9ssssssssENDsssss  
0FssssssTAPEsssss is written on the tape. (Here s represents  
blanks).

USAGE:

1. Calling Sequence:

This routine is used with the following FORTRAN calling  
sequences:

CALL SAVF (N), or

DUMMY = SAVF (N)

where N is the number, 1-48, of the logical tape to be saved.

TDR-63-11



7. PROGRAM LISTING



```

*** SPURT ***
SEPTEMBER 19, 1962 MOD OF AUG. 21, 1962
IF KNTRL (1)=1, USE METRO DATA
IF KNTRL (1)=2, USE WINDS ONLY
IF KNTRL (2)=1, PRINT INPUT DATA
IF KNTRL (3)=N, WRITE OUTPUT FOR BATT ON TAPE N
IF KNTRL (4)=1, USE TWOBODY FOR LAST STAGE ONLY
IF KNTRL (4)=2, USE TWOBODY FOR ALL STAGES
IF KNTRL (5)=NO NO IS NUMBER OF LINES ON OUTPUT PAGE
IF KNTRL (6)=N, PLOT TAPE IS WRITTEN ON TAPE N
IF KNTRL (7)=1, A RIGHT HAND COORDINATE SYSTEM
IS PRINTED OUT
IF KNTRL (8)=1, USE IMPACT FOR N-1 STAGES
IF KNTRL (8)=2, USE IMPACT FOR ALL STAGES
SENSE SWITCH 4 PRINTS INTERMEDIATE RESULTS
SENSE SWITCH 5 REINITIALIZES ON CRITERION FAILURE
A NEG NO. IN FIRST POSITION OF BURNING DRAG-MACH NO.
TABLE IGNORES AERODYNAMICS FOR THAT STAGE

```

\*\*\* DIMENSION BLOCK \*\*\*

```

DIMENSION NAME(10),KNTRL(10),TMCC(10),SPTIME(100),SPIN(100),TIMET
1(21,10),THRUST(21,10),DIMACH(21,10),DRAG1(21,10),D2MACH(21,10),DRA
2G2(21,10),CPMACH(21,10),CP(21,10),CNMACH(21,10),CN(21,10),FTT(10),
3AE(10),PAT(10),D(10),GO(10),GP(10),R(10),A(10),B(10),PWGT(10),PWGT
4C(10),TMI(10),TMB0(10),WTEMP(96),WDEN(96),WPRES(96),WINDV(96),WIND
5A(96),WGT(10),WGTC(10),WALTU(98),DC(3,3),ACODES(10),PXLDD(3)
6,SPT(100),SPI(100),WALT(96),WTU(96),WPU(96),
7STX(3),STXD(3),PXD(3),WX(98),WY(98),WZ(98),ROT(3,3),RA(10),
8RB(10),TTTAB(21),TTTAB(21),DMTAB(21),DTAB(21),CPMTAB(21),CPTAB(21)
9CNMTAB(21),CNTAB(21),WGTTAB(21),SPIT(100),GOP(10),N(6,10)
DIMENSION ROT1(3,3),T(3),T(3),TDEL(10),PHT(10),PXDD(3)
1,OUT(25,50),PXL(3),PXLDD(3),PXND(3),XVX(3),BL(38),TABLE(500),CBLOCK
2(150),VX(3),TB3(5),TB4(5),TB6(3,50),TB7(50),TB8(50),TB9(50)
3XMACH(10,10),CDM(10,10),NNX(10),XIMPA(3,10),XDIMP(3,10),TFIMP(10
4),RANGE(10),DELPR(10)

```

\*\*\* COMMON BLOCK \*\*\*

```

COMMON ROT1,NAME,KBATT,JTB1,TB2,TB3,TB4,JTB5,TB6,TB7,TB8,TB9,Z2
1,STHC,CTHC,ALO2,REL,XMACH,CDM,NNX,NOT,RANGE,XIMPA,XDIMP,TFIMP,DELP
2R

```

[illegible]

```

17 FORMAT (1H0F10.4,9E12.3/(5X10F11.3))
18 FORMAT(2H1 10A8)
20 FORMAT(/19X23HLOCAL FLIGHT PARAMETERS/3X4HTIME4X8HLATITUDE4X9HLONG
11TUDE3X8HALTITUDE2X8HVELOCITY2X7HAZIMUTH3X3HFPA/4X3HSEC7X3HDEG10X3
2HDEG7X4HFEET4X6HFT/SEC6X3HDEG5X3HDEG/)
21 FORMAT(/83H TIME RANGE DEFLECTION ALTITUDE VELOCITY
1 PHI THETA FT/SEC DEG. DEG. N.MILES SEC N.MILES N.MILES
2 N.MILES /)
22 FORMAT(/3X4HTIME5X6HTRUST5X6HWEIGHT4X12HTOTAL ACCEL.4X12HDYNAMIC
1PRES4X5H DRAG7X5H MACH/4X3HSEC7X3HLBS8X3HLBS5X12HACCEL./GRAV.6X8HL
2BS/SQFT7X3HLBS9X3HNO./)
23 FORMAT(/4X4HTIME9X1HX12X1HY12X1HZ10X5HX-DOT8X5HY-DOT8X5HZ-DOT/4X3H
1SEC9X4HFEET9X4HFEET9X4HFEET8X6HFT/SEC7X6HFT/SEC7X6HFT/SEC/)
24 FORMAT(F8.2,2F12.4,F9.0,F9.0,2F9.2)
25 FORMAT(F8.2,F10.3,2F12.3,F11.1,2F8.3)
26 FORMAT(F8.2,2F12.2,2F12.3,2F13.2)
27 FORMAT(F8.2,6F13.0)
30 FORMAT(11,F10.6)
31 FORMAT(2F10.0)
32 FORMAT(13)
33 FORMAT(3A8,3F10.0)
ORG 100508

```

\*\*\* CONSTANTS \*\*\*

```

PI=3.141592654
TWOPI=2.*PI
HALFPI=0.5*PI
RAD=PI/180.
REE=20925647.
GM=1.40764E16
XJ=.0016234
GRAV0=32.174
OMEG=7.292111E-5
TOMEG=2.*OMEG
OMEG2=OMEG*OMEG
GK= XJ*REE*REE
ICODE=3
TRERR=1.E-3
ACODE=1.0
ACODES(1)=1.0

```

```

ACODES(2)=1.0
ACODES(3)=1.0
ACODES(4)=0.1
ACODES(5)=0.1
ACODES(6)=1.0
ACODES(7)=1.0
ACODES(8)=1.0
ACODES(9)=0.01
ACODES(10)=0.01

```

```

DT=1.
SMIN=.01
SMAX=1.
NOE=10
SS=1./16.

```

\*\*\* INPUT BLOCK \*\*\*

```

100 CALL SCLOCK
    KIX = 0
    READ 3,NS,PYLWGT
    IF(NS)120,110,140
    IF(NOT) 111,112,111
110 CALL SAVF(NOT)
111 STOP
112
120 NS=NS
    ENA 0
    ENG 0
    RTJ RDF
    NOP =49
    RTJ ERROR.
130 GO TO 183
140 READ 1,(NAME(I),I=1,10)
    READ 5,ALAT,ALON,AALT,AAZIM,(STX(I),I=1,3),(SIXD(I),I=1,3),STP,STT
    1,STPD,STTD,STARTT,TSTOP
    READ 2,(KNTRL(I),I=1,10),NSPIN,NOT,INTN
    IF(NOT) 113,114,113
113 REWIND NOT
114 READ 5,(SPTIME(I),I=1,NSPIN)
    READ 5,(SPIN(I),I=1,NSPIN)
    DO 150 J=1,NS
    READ 6,NO,(N(I,J),I=1,5)
    NI=N(1,J)

```

```

N2=N(2,J)
N3=N(3,J)
N4=N(4,J)
N5=N(5,J)
READ 5,{TIMET(I,J),I=1,N1}
READ 5,{THRUST(I,J),I=1,N1}
READ 5,{DIMACH(I,J),I=1,N2}
IF (DIMACH(1,J)) 142,141,141
141 READ 5,{DRAG1(I,J),I=1,N2}
READ 5,{D2MACH(I,J),I=1,N3}
READ 5,{DRAG2(I,J),I=1,N3}
READ 5,{CPMACH(I,J),I=1,N4}
READ 5,{CP(I,J),I=1,N4}
READ 5,{CNMACH(I,J),I=1,N5}
READ 5,{CN(I,J),I=1,N5}
142 READ 5,PAT(J),AE(J),D(J),GO(J),GP(J),RA(J),RB(J),A(J),B(J),PWGT(J)
1,PWGT(J),PHT(J),TDEL(J),DUM,TMT(J),TMO(J),TMCC(J)
IF(NO-J)149,150,149
149 PRINT 11,J
STOP
150 CONTINUE
IF(KNTRL(1)) 190,190,160
160 READ 2,KMETRO
READ 5,{WALT(I),I=1,KMETRO}
IF(KNTRL(1)-1)190,170,180
170 READ 5,{WTEMP(I),I=1,KMETRO}
READ 5,{WDEN(I),I=1,KMETRO}
READ 5,{WPRES(I),I=1,KMETRO}
READ 5,{WINDV(I),I=1,KMETRO}
READ 5,{WINDA(I),I=1,KMETRO}
190 IF(KNTRL(4)) 188,188,187
187 READ 30,JTB1,TB2
READ 31,{TB3(I),TB4(I),I=1,JTB1}
READ 32,JTB5
DO 189 I=1,JTB5
189 READ 33,{TB6(I,I),I=1,3},TB7(I),TB8(I),TB9(I)
188 IF(KNTRL(8)) 183,183,184
184 READ 6,NXIM
DO 185 J=1,NXIM
READ 3,NRAT,DELPR(J)
NNX(J)=NRAT
READ 5,{XMACH(I,J),I=1,NRAT}

```



```

185 READ 5,( CDM(I,J),I=1,NRAT)
186 CONTINUE
187 IF (KNTRL(2)) 200,200,192
188 WRITE OUTPUT TAPE NOT,7,(NAME(I),I=1,10)
189 WRITE OUTPUT TAPE NOT,8,PYLWGT,ALAT,ALON,AALT,AAZIM,(KNTRL(I),I=1,
190 110)
191 WRITE OUTPUT TAPE NOT,10,(SPTIME(I),SPIN(I),SPTIME(I+25),SPIN(I+25
192 1),SPTIME(I+50),SPIN(I+50),SPTIME(I+75),SPIN(I+75),I=1,25)
193 DO 191 J=1,NS
194 WRITE OUTPUT TAPE NOT,9,J
195 WRITE OUTPUT TAPE NOT,12,(TIMET(I,J),THRUST(I,J),DIMMACH(I,J),DRAG1
196 1(I,J),D2MACH(I,J),DRAG2(I,J),CPMACH(I,J),CP(I,J),CNMACH(I,J),CN(I,
197 2J),I=1,21)
198 WRITE OUTPUT TAPE NOT,13,PWGT(J),PWGTC(J),TMI(J),TMO(J),GO(J),
199 2GP(J),AE(J),PAT(J),A(J),B(J),RA(J),RB(J)
200 WRITE OUTPUT TAPE NOT,14,IDEL(J),PHT(J),D(J),TMCC(J)
201 *** GEODETIC TO GEOCENTRIC ***
202 ALA2=ALAT*RAD
203 AL02=ALON*RAD
204 AAZ12=AAZIM*RAD
205 COLAT=HALFPI-ALA2
206 BLON=AL02+HALFPI
207 BAZIM=AAZ12-HALFPI
208 CALAT=COSF(ALA2)
209 SILAT=SINF(ALA2)
210 C=REE/SQRTF(1.-.006693421*SILAT*SILAT)
211 S=.99330658*C
212 X1X=(C+AALT)*CALAT
213 X1Y=(S+AALT)*SILAT
214 REL=SQRTF(X1X*X1X+X1Y*X1Y)
215 THC=ATANF(X1Y/X1X)
216 STHC = SINF (THC)
217 COTHC=HALFPI-THC
218 CTHC = COSF (THC)
219 CALL ROTATE(BAZIM,DELTH,0.,STX,XVX)
220 XVX(3)=XVX(3)+REL
221 CALL ROTATE(0.,-COTHC,-BLON,XVX,PX)
222 DELTH=ALA2-THC
223 CALL ROTATE (BAZIM,-COLAT,-BLON,STXD,PXD)
224 DO 201 I=1,3

```

```

201 DO 201 J=1,3
    ROT(I,J)=ROT1(I,J)
    Z2=KNTRL(5)
    PHI=STP*PI
    THETA=STT*PI
    R1X=PXD(1)
    R2X=PXD(2)
    R3X=PXD(3)
    R4X=STPD*PI
    R5X=STTD*PI
    LSKIP=0
    JP=1
    L=0
    TIME = STARTT
    KBATT = 0
    KPLOT = 0
    IF (KNTRL(3)) 202,203,202
202 KBATT = KNTRL(3)
    REWIND KBATT
203 IF (KNTRL(6)) 204,205,204
204 KPLOT = KNTRL(6)
    REWIND KPLOT

    *** SET UP INPUT TABLES ***

205 WGT(NS+1)=PYLWGT/GRAVO
    DO 210 J=1,NS
        K=NS-J+1
        WGT(K)=PWGT(K)/GRAVO+WGT(K+1)
        WGT(J)=PWGT(J)/GRAVO
210 GOP(J)=GO(J)-GP(J)
    DO 220 I=1,NSPIN
        K=NSPIN-I+1
        SPT(K)=SPTIME(I)
220 SPI(K)=SPIN(I)
        SPIT(NSPIN)=0.
    DO 221 I=2,NSPIN
        K=NSPIN-I+1
221 SPIT(K)=SPIT(K+1)+(SPI(K)+SPI(K+1))*(SPT(K)-SPT(K+1))/2.
        IF (KNTRL(1)) 229,229,230
229 WMAX = 0.
    GO TO 260

```

```

230 DO 251 I=1,KMETRO
    K = KMETRO-I+1
    WALTU(K)=WALT(I)
    IF (KNTRL(1)-1) 260,240,250
240 WTU(K)=1116.4*SQR((WTEMP(I)+273.16)/288.16)
    WDU(K)=WDEN(I)*.00194032
    WPU(K)=WPRES(I)*2.088544
250 SW=WINDA(I)*RAD-BAZIM+HALFPI
    VMX=COSF(SW)
    VMY=-SINF(SW)
    WX(K)=WINDV(I)*{ROT(1,1)*VMX+ROT(1,2)*VMY}
    WY(K)=WINDV(I)*{ROT(2,1)*VMX+ROT(2,2)*VMY}
    WZ(K)=WINDV(I)*{ROT(3,1)*VMX+ROT(3,2)*VMY}
    WMAX = WALTU(1)
260 CALL SETTAB(NS,TMI,TMCC,TMBO,DT,TABLE,IT)
    GO TO 281
270 IF (TIME-TMBO(L)) 410,271,273
271 DO 272 I=1,N3
    K=N3-I+1
    DMTAB(K)=D2MACH(I,L)
    DTAB(K)=DRAG2(I,L)*D(L)*PI/8.
272 DMTAB(1)=10.E10
    DTAB(1)=DTAB(2)
    BDOT=0.
    FT=0.
    WGTCI=WGTC(L)
    SENSE LIGHT 1
    IF (KNTRL(4)-2) 280,279,280
279 CALL TW080D (TIME,PX,PXD)
280 KIX=KIX+1
    DO 274 I=1,3
    XIMPA(I,KIX)=PX(I)
    XDIMP(I,KIX)=PXD(I)
    TFIMP(KIX)=TIME
274 TFIMP(KIX)=RANGE1
    RANGE(KIX)=RANGE1
273 IF (TIME-TMCC(L)) 999,281,410
281 L=L+1
    WGTCI=0.
    GI=GO(L)
    N1=N(1,L)
    N2=N(2,L)+1
    N3=N(3,L)+1

```

```

N4=N(4,L)+1
N5=N(5,L)+1
P1=PAT(L)*AE(L)
DO 400 I=1,N1
K=N1-I+1
TTTAB(K)=TIMET(I,L)+TMI(L)
THTAB(K)=THRUST(I,L)+P1
WGTAB(N1)=0.
DO 404 I=2,N1
K=N1-I+1
WGTAB(K)=WGTAB(K+1)+(THTAB(K)+THTAB(K+1))*(TTTAB(K)-TTTAB(K+1))*0.
15
FTT(L)=WGTAB(1)
DO 405 I=1,N1
WGTAB(I)=WGTAB(I)*WGTC(L)/FTT(L)
IF(D1MACH(1,L))406,407,407
406 LSKIP=1
GO TO 410
407 LSKIP=0
DO 401 I=1,N2
K=N2-I+1
DMTAB(K)=D1MACH(I,L)
DTAB(K)=DRAG1(I,L)*D(L)*D(L)*PI/8.
DO 402 I=1,N4
K=N4-I+1
CPMTAB(K)=CPMACH(I,L)
CPTAB(K)=CP(I,L)
DO 403 I=1,N5
K=N5-I+1
CNMTAB(K)=CNMACH(I,L)
CNTAB(K)=CN(I,L)*PI/8.
DMTAB(1)=10.E10
DTAB(1)=DTAB(2)
CPMTAB(1)=10.E10
CPTAB(1)=CPTAB(2)
CNMTAB(1)=10.E10
CNTAB(1)=CNTAB(2)
410 IF(TIME-TMI(L))999,411,411
411 SENSE LIGHT 0
IF(TIME)412,412,999
412 CHECK=PX(1)
RTJ TEXTIT

```

	CALLING	SEQUENCE
(999)	ADAMS EQN TEXTIT PRTIME BL CBLOCK EXIT TMIN ERR STA PRINT 15, ERR STOP	INTEGRATION
C		*** EQUATIONS OF MOTION ***
C		
C		
EQN	SLJ PXD(1)=R1X PXD(2)=R2X PXD(3)=R3X PHID=R4X THETD=R5X R2=PX(1)*PX(2)+PX(2)*PX(3)+PX(3) R=SQRTF(R2) RE=REE/SQRTF(1+.00673852*PX(3)/R*PX(3)/R) ALT=R-RE IF(LSKIP)1050,1001,1050 IF(ALT-WMAX)1000,1030,1030 IF(KNTRL(1)-1)1030,1010,1020 RHO = INTERPF(ALT,KMETRO,INTN,WALTU,WDU) VA = INTERPF(ALT,KMETRO,INTN,WALTU,WTU) PRES = INTERPF(ALT,KMETRO,INTN,WALTU,WPU) 1020 VWX = INTERPF(ALT,KMETRO,INTN,WALTU,WX) VWY = INTERPF(ALT,KMETRO,INTN,WALTU,WY) VWZ = INTERPF(ALT,KMETRO,INTN,WALTU,WZ) IF (KNTRL(1)-1) 1031,1040,1031 1030 VWX = 0. VWY = 0. VWZ = 0.	
	ALT ATMOS	ATMOSPHERE
(1031)	LDA RTJ OCT OCT OCT	CALL ROUTINE
TEM		
PRES		
RHO		

```

VA      OCT
1040    GO TO 1050
        VX{1} = PXD{1}-VWX
        VX{2} = PXD{2}-VMY
        VX{3} = PXD{3}-VWZ
        V = RADF(VX)
        VMACH=V/VA
        FDRAG=INTERPF(VMACH,N2,INTN,DMTAB,DTAB)*V*V*RH0
        CPI=INTERPF(VMACH,N4,INTN,CPMTAB,CPTAB)
        CNI=INTERPF(VMACH,N5,INTN,CNMTAB,CNTAB)
        GO TO 1060
1050    FDRAG=0.
        CPI=0.
        CNI=0.
        VMACH=0.
        RH0=0.
1060    IF(SENSE LIGHT 1)1061,1062
1061    SENSE LIGHT 1
        GO TO 1063
1062    FORCE=INTERPF(TIME,N1,INTN,TTTAB,THTAB)
        WGTCI=INTERPF(TIME,N1,INTN,TTTAB,WGTAB)
        FT=FORCE-PRES*AE{L}/144.
        BDOT=-FORCE*WGTC(L)/FTT(L)*(WGT(L)*GOP(L)/WGTI*WGT(L)*GOP(L)/WGTI+
        1RB(L))
1063    GRV=GM/(R2*R)*(1.+3.*GK/R2-15.*GK*PX(3)/R2*PX(3)/R2)
        WGTI=WGT(L)-WGTCI
        GI=GO(L)+GOP(L)*WGTCI/WGTI
        GRVX=-GRV*PX(1)
        GRVY=-GRV*PX(2)
        GRVZ=-(GRV+6./R*GK/R2*GM/R2)*PX(3)
        IF(TIME-TMI(L))1082,1080,1082
1080    AXLM=A(L)
        TRVM=B(L)
        GO TO 1083
1082    AXLM=A(L)-RA(L)*WGTCI
        TRVM=B(L)-WGT(L)*(GI-GO(L))*GOP(L)-WGTCI*RB(L)
1083    BDOTB=BDOT/TRVM
        AXLMB=AXLM/TRVM
        COEF=CNI*(CPI-GI)*D(L)*D(L)*V*RH0/TRVM
        XSP=INTERPF(TIME,NSPIN,INTN,SPT,SPI)
        XSPT=MODF(INTERPF(TIME,NSPIN,INTN,SPT,SPT),TWOPI)
        SPHI=SINF(PHI)

```

```

CPHI=COSF(PHI)
STHET=SINF(THETA)
CTHET=COSF(THETA)
TPHO=PHI(L)+XSPT
CPHT=COSF(TPHO)
SPHT=SINF(TPHO)
IF(LSKIP)1091,1089,1091
1089 DO 1090 I=1,3
      T(I)=0.
DO 1090 J=1,3
      T(I)=ROT(J,I)*VX(J)+T(I)
      V2=-T(1)*CPHI+T(3)*SPHI
      V3=-T(1)*SPHI*STHET-T(2)*CTHET-T(3)*CPHI*STHET
1091 T(1)=FT*(CTHET*SPHI-TDEL(L)*(CPHT*CPHI+SPHT*SPHI*STHET))
      T(2)=-FT*(STHET+TDEL(L)*SPHT*CTHET)
      T(3)=FT*(CTHET*CPHI+TDEL(L)*(CPHT*SPHI-SPHT*CPHI*STHET))
DO 1100 I=1,3
      TT(I)=0.
DO 1100 J=1,3
      TT(I)=ROT(I,J)*T(J)+TT(I)
      H = RADF(PXD)
1101 PXDD(1)={TT(1)-FDRAG*PXD(1)/H}/WGTT+GRVX+TOMEG*PXD(2)+OMEG2*PX(1)
      PXDD(2)={TT(2)-FDRAG*PXD(2)/H}/WGTT+GRVY-TOMEG*PXD(1)+OMEG2*PX(2)
      PXDD(3)={TT(3)-FDRAG*PXD(3)/H}/WGTT+GRVZ
      PHIDD=-BDOTB*PHID+((2.-AXLMB)*STHET*PHID+AXLMB*XSP)*THE TD+COEF*V2
      1+FT*GI*TDEL(L)*CPHT/TRVM)/CTHET
      THETDD=-BDOTB*THE TD-((1.-AXLMB)*STHET*PHID+AXLMB*XSP)*PHID*CTHET-
      1T*GI*TDEL(L)*SPHT/TRVM-COEF*V3
      GO TO EQN
EXIT SLJ **
502 CHECK=PX(1)
IF(SENSE SWITCH 4) 500,505
500 PRINT 17, TIME,(PX(1),I=1,3),(PXD(1),I=1,3),(PXDD(1),I=1,3),PHI,TH
      IETA,PHID,THETD,PHIDD,THETDD,ALT,V,VMACH,FT,GRVX,GRVY,GRVZ,WGTI,FDR
      ZAG,CPI,CNI,AXLM,TRVM,BDOT
505 IF(SENSE LIGHT 2) 999,EXIT
      TMIN SLJ **
PRINT 16,TIME
IF (SENSE SWITCH 5) 550, TMIN
550 SENSE LIGHT 2
      GO TO TMIN
TEXT17 SLJ **

```

```

(3999) RTJ EQN
C
C
C
4000 DO 4001 I=1,3
      PXLD(I)=0.
      DO 4001 J=1,3
        PXLD(I)=ROT(J,I)*PXD(J)+PXLD(I)
        CALL ROTATE(BLON,COTH,0.,PX,XVX)
        XVX(3)=XVX(3)-REL
        CALL ROTATE(0.,-DELTH,-BAZIM,XVX,PXL)
        OUT(1,JP)=TIME
        TP1=RADF(PX)
        TP=ASINF(PX(3)/TP1)
        CALL GEODED(TP,TP1,TP2,OUT(4,JP))
        OUT(2,JP)=TP2/RAD
        OUT(3,JP)=ATANF(PX(2)/PX(1))/RAD
        IF(PX(1)) 4010,4011,4011
        OUT(3,JP)=OUT(3,JP)+180.
        AN1=OUT(3,JP)*RAD+HALFPI
        AN2=HALFPI-TP2
        CALL ROTATE(AN1,AN2,0.,PXD,PXND)
        OUT(5,JP)=RADF(PXND)
        IF(TIME) 4015,4015,4016
        OUT(6,JP)=AAZIM
        OUT(7,JP)=(HALFPI-PHI)/RAD
        GO TO 4019
      4016 OUT(6,JP)=ATANF(PXND(1)/PXND(2))/RAD
        IF(PXND(2)) 4020,4021,4021
        OUT(6,JP)=OUT(6,JP)+180.
        OUT(7,JP)=ASINF(PXND(3)/OUT(5,JP))/RAD
        4021 IF(TIME-12.) 4023,4023,4022
        4019 AN1 = SINF((THC+TP)/2.)
        OUT(8,JP)=REE/6076.1033*ACOSF(SINHC*PX(3)/TP1+CTHC*COSEF(TP)*COSEF(
        1AL02-OUT(3,JP)*RAD))/SQRTF(1.-.00673852*AN1*AN1)
        GO TO 4024
      4023 OUT(8,JP)=PXL(1)/6076.1033
      4024 OUT(9,JP)=PXL(2)/6076.1033
        RANGE1=OUT(8,JP)
        OUT(10,JP)=OUT(4,JP)/6076.1033
        OUT(11,JP)=RADF(PXLD)

```



```

OUT(12,JP)=PHI/RAD
OUT(13,JP)=THETA/RAD
OUT(14,JP)=FT
OUT(15,JP)=WGTI*GRAVO
OUT(16,JP)=RADF(PXDD)/GRAVO
OUT(17,JP)=.5*RHO*OUT(5,JP)*OUT(5,JP)
OUT(18,JP)=FDRAG
OUT(19,JP)=VMACH
OUT(20,JP)=PXL(1)
OUT(21,JP)=-PXL(2)
OUT(22,JP)=PXL(3)
OUT(23,JP)=PXL(1)
OUT(24,JP)=-PXL(2)
OUT(25,JP)=PXL(3)
IF (KNTRL(7)) 40241,40241,40242
40242 OUT(21,JP)=-OUT(21,JP)
OUT(24,JP)=-OUT(24,JP)
40241 IF (KNTRL(6)) 4026,4026,4025
4025 CALL WOTF (OUT(1,JP),OUT(25,JP),KPLLOT,0)
4026 IF (KNTRL(3)) 4029,4029,4027
4027 AN1 = HALFPI-PHI
AN2 = HALFPI+THETA
CALL ROTATE (HALFPI,AN1,AN2,T,TT)
DO 4028 I=1,3
  PXLDD(1)=0.
DO 4028 II=1,3
  DC (I,II)=0.
  PXLDD(1) = ROT(II,1)*PXDD(II)+PXLDD(I)
DO 4028 III=1,3
  DC(1,II)=ROT(I,III)*ROT1(III,II)+DC(1,II)
  WRITE TAPE KBATT (OUT(1,JP),I=1,25),(PXLDD(1),I=1,3),
1((DC(1,II),I=1,3),II=1,3),(PX(I),I=1,3),(PXD(1),I=1,3)
4029 IF (JP-KNTRL(5)) 4030,4031,4031
4030 IF (TIME-TSTOP)4059,4031,4031
4031 WRITE OUTPUT TAPE NOT,18,(NAME(I),I=1,10)
WRITE OUTPUT TAPE NOT,20
WRITE OUTPUT TAPE NOT,24,((OUT(1,II),I=1,7),II=1,JP)
WRITE OUTPUT TAPE NOT,18,(NAME(I),I=1,10)
WRITE OUTPUT TAPE NOT,21
WRITE OUTPUT TAPE NOT,25,(OUT(1,II),I=8,13),II=1,JP)
WRITE OUTPUT TAPE NOT,18,(NAME(I),I=1,10)
WRITE OUTPUT TAPE NOT,22

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```

WRITE OUTPUT TAPE NOT,26,{OUT(1,11),(OUT(1,11),I=14,19),I1=1,JP)
WRITE OUTPUT TAPE NOT,18,{NAME(I),I=1,10}
WRITE OUTPUT TAPE NOT,23
WRITE OUTPUT TAPE NOT,27,{OUT(1,11),(OUT(1,11),I=20,25),I1=1,JP)
JP=1
4036 IF (TIME-TSTOP) 4060,4037,4037
4037 IF (KNTRL(3)) 4039,4039,4038
4038 END FILE KBATT
4039 IF (KNTRL(6)) 4040,4040,40391
40391 END FILE KPLOT
4040 IF (KNTRL(8)-2) 4071,4070,4071
4070 KIX = KIX + 1
DO 4073 I = 1,3
XIMPA(I,KIX) = PX(I)
XDIMP(I,KIX) = PXD(I)
TFIMP(KIX) = TIME
RANGE(KIX) = RANGE1
4071 IF (KNTRL(8)) 5010,5010, 5011
5011 DO 5009 I=1,KIX
IRA=I
CALL IMPACT (IRA)
CONTINUE
5009 IF (KNTRL(4)) 4041,4050,4041
5010 CALL TWO80D (TIME,PX,PXD)
4041 IF (KNTRL(3)) 4052,4052,4051
4050 CALL SAVF (KBATT)
4051 IF (KNTRL(6)) 4054,4054,4053
4052 CALL SAVF (KPLOT)
4053 CALL ECLOCK(NOT)
4054 GO TO 100
4059 JP=JP+1
4060 IT=IT-1
PRIME=TABLE(IT)
IF(MOCF(TABLE(IT+1),OT))270,TEXIT,270
RDF LIB RDF
END
C
C

```

```

SUBROUTINE SETTAB (NS,TMI,TMCC,TMBO,DT,TABLE,IX)
THIS SUBROUTINE SETS UP THE PRINT TIME TABLE
DIMENSION TMI(10),TMCC(10),TMBO(10),TABLE(500)
NSS=NS-1
IX=(TMBO(NS)+1.)/DT
TABLE(IX)=0.
DO 1 I=2,IX
K=IX-I+1
TABLE(K)=TABLE(K+1)+DT
DO 2 I=2,NSS
U=TMI(I)
RTJ INS
TMI(I)=U
DO 3 I=1,NSS
U=TMBO(I)
RTJ INS
TMBO(I)=U
DO 4 I=1,NSS
U=TMCC(I)
RTJ INS
TMCC(I)=U
RETURN
DUM=MODF(DUM,1.)
SLJ **
SIL 6 INS3
LDA U
LIL 6 IX
THS 6 TABLE
INI 6 -1
INI 6 1
FSB 6 TABLE
AJP 1 INS1
LDA 6 TABLE
LDQ DT
RTJ MODF
RTJ ERROR.
AJP N **3
LDA 6 TABLE
FAD INS3+1
STA 6 TABLE
STA U
LIL 6 INS3

```

INS1	SLJ	INS
	SIU	6 #+3
	LIL	6 IX
	LDA	6 TABLE
	INI	6 1
	STA	6 TABLE
+	ISK	6 **
	SLJ	INS2
	LIU	6 #-1
	LDA	U
	STA	6 TABLE
	RAO	IX
	LIL	6 INS3
INS2	SLJ	INS
	INI	6 -3
INS3	SLJ	INS1+1
	ZRO	
	DEC	.0001
	END	

```

SUBROUTINE ECLOCK(ITAPE)
  THIS PROGRAM COMPUTES THE MACHINE TIME FOR ONE TRAJECTORY RUN
  DIMENSION X(3)
  EXF 0 02000B
  LDA 0
  SCM MASK1
  FAD ZERO
  FDV F216M
  STA X+1
  ENA X+1
  RTJ INTF
  NOP
  STA X
  LDA X+1
  FSB X
  FMU F60
  STA X+2
  ENA X+2
  RTJ INTF
  NOP
  STA X+1
  LDA X+2
  FSB X+1
  FMU F60
  STA X+2
  NOP
  WRITE OUTPUT TAPE ITAPE,2,(X(1), I=1,3)
  FORMAT(16HOELAPSED TIME = F3.1,7H HOURS F4.1,10H MINUTES F6.3,8H
  1SECONDS)
  RETURN
MASK1 OCT 2044000000000000000
F60 DEC 60.
F216M DEC 216000.
ZERO DEC 0
INTF LIB INTF
END

```

```

SUBROUTINE SCLOCK
  EXF 0 020008
  ENA 0
  STA 08
  EXF 0 010008
  RETURN
END

```

STOP THE CLOCK

START THE CLOCK AFTER SETTING=0

+

-

```

C
C
SUBROUTINE TWOBOD (TBO,A1,A2)
  THIS SUBROUTINE COMPUTES A KEPLERIAN TRAJECTORY ALONG WITH
  LOOK ANGLES FROM VARIOUS STATIONS
  DIMENSION NAME(10),ANS(5),PRTIME(2000),THETA(2000),PHI(2000),
1ALT(2000),DELTAT(5),ENDT(5),TAG(3,50),SX(3),FX(3),XSTH(50)
2,DUMTB(9),A1(3),A2(3),XSPH(50),XSHT(50),A1A(10,10),A1B(10,10),N1C(
310)
  COMMON DUMTB,NAME,KBATT,NOT,ORBT,DELTAT,ENDT,NOST,TAG,XSTH,
1XSPH,XSHT,Z2,SLAL,CLAL,XLLO,RLAU,A1A,A1B,N1C,NAT
  SECF(X)=MODF(X,60.)
  FMINF(X)=MODF(INTF(X/60.),60.)
  HOURF(X)=INTF(X/3600.)
  REE=3443.9255
  WE=7.292115E-5
  E=.08181333
  GM=1.4076427E+16
  GM1=GM/6076.1033
  P1=3.141592654
  RAD=PI/180.0
  F=1.0/298.3
  ER=0.000005
  POP=E*E/(1.0-E*E)
  SLLO=SINF(XLLO)
  CLLO=COSF(XLLO)
  XLAU=CLAL*CLLO
  YLAU=CLAL*SLLO
  ZLAU=SLAL
  AX=A1(1)
  AY=A1(2)
  AZ=A1(3)
  AXDOT=A2(1)-AY*WE
  AYDOT=A2(2)+AX*WE
  AZDOT=A2(3)
  RF=SQRTF(AX*AX+AY*AY+AZ*AZ)
  R=RF/6076.1033
  VI=SQRTF(AXDOT*AXDOT+AYDOT*AYDOT+AZDOT*AZDOT)
  H=R*VI*VI/GM1
  IF(H-2.0)104,110,110
102 WRITE OUTPUT TAPE NAT,200,(NAME(I),I=1,9)
110 WRITE OUTPUT TAPE NAT,202
  GO TO 190
104 SMAJ=R/(2.0-H)

```

```

HX=AY*AZDOT-AZ*AYDOT
HY=AZ*AXDOT-AX*AZDOT
HZ=AX*AYDOT-AY*AXDOT
HH=HX*HX+HY*HY+HZ*HZ
H1=SQRTF(HH)
SLANT =ACOSF(HZ/H1)
CAPO=ATANF(-HX/HY)
IF(HY)106,106,105
CAPO=PI+CAPO
105
106 RDOT=(AX*AXDOT+AY*AYDOT+AZ*AZDOT)/RF
EX=SQRTF(1.0-H1*H1/(SMAJ*GM*6076.1033))
CTRUA=H1/RF*H1/GM-1.0
STRUA=H1*RDOT/GM
TRANOM=ATANF(STRUA/CTRUA)
IF(CTRUA)111,112,112
111 TRANOM=PI+TRANOM
112 CMU=(AY*HX-AX*HY)/H1
AMU=ATANF(AZ/CMU)
IF(CMU)1120,1121,1121
1120 AMU=PI+AMU
1121 SMOM=AMU-TRANOM
SMIN=SMAJ*SQRTF(1.0-EX*EX)
PERIOD=4.184624E-4*SMAJ**1.5
ETA=PERIOD*60./(2.*PI)
APOGEE=SMAJ*(1.0+EX)-REE
PERGEE=SMAJ*(1.0-EX)-REE
XX=SQRTF((1.0-EX)/(1.0+EX))
EXANOM=2.0*ATANF(XX*TANF(TRANOM/2.0))
TOPP=(EXANOM-EX*SINF(EXANOM))*ETA
WRITE OUTPUT TAPE NAT,200,(NAME(I),I=1,9)
ANS(1)=SLANT/RAD
ANS(2)=SMOM/RAD
ANS(3)=CAPO/RAD
ANS(4)=TRANOM/RAD
ANS(5)=EXANOM/RAD
WRITE OUTPUT TAPE NAT,201,SMAJ,SMIN,EX,PERIOD,APOGEE,PERGEE,(ANS(I
1),I=1,5)
C
C TRAJECTORY POINTS
IF(PERGEE)1123,1124,1124
1123 TRIMP=ACOSF(((1.-EX*EX)*SMAJ/REE-1.)/EX)
EXIMP=2.*PI-2.*ATANF(XX*TANF(TRIMP/2.))

```



```

TIMP=(EXIMP-EX*SINF(EXIMP))*ETA
GO TO 1127
1124 IF(ORBT)1125,1125,1126
1125 TIMP=PERIOD*60.
GO TO 1127
1126 TIMP=ORBT*PERIOD*60.
1127 SSLANT=SINF(SLANT)
CSLANT=COSF(SLANT)
E1=EXANOM
TIME=TOPP
PN=-1.0
J=0
TINCR=0.0
MEX=0
MM=1
GO TO 117
TINCR=60.*(INTF(TBO/60.)+1.)-TBO
MM=2
GO TO 117
MM=3
MEX=MEX+1
TINCR=DELIAT(MEX)
1116 IF(TIME+TBO-TOPP-ENDT(MEX)+.01)117,115,115
1117 J=J+1
TIME=TIME+TINCR
IF(TIME-TIMP)118,118,1171
TIME=TIMP
XM=TIME/ETA
E2=E1-(E1-EX*SINF(E1)-XM)/(1.0-EX*COSF(E1))
1119 IF(ABSF(E2-E1)-ER) 121,121,120
E1=E2
GO TO 119
TRUNU=2.0*ATANF(TANF(E2/2.0)/XX)
R=SMAJ*(1.0-EX*EX)/(1.0+EX*COSF(TRUNU))
VELOC=SQRTF(GMI*(2./R-1./SMAJ))
FPA=ATANF(EX*SINF(TRUNU)/(1.0+EX*COSF(TRUNU)))/RAD
SX(1)=COSF(TRUNU)
SX(2)=SINF(TRUNU)
SX(3)=0.0
CALL ROTATE(-SMOM,-SLANT,-CAPO,SX,FX)
THETA(J)=ATANF(FX(3)/SQRTF(FX(1)*FX(1)+FX(2)*FX(2)))
PHIX=ATANF(FX(2)/FX(1))

```

```

122 IF(FX(1))122,123,123
123 PHIX=PI+PHIX
    PHIX=PHIX-WE*(TIME-TOPP)
    PHI(J)=MODF(PHIX,2.*PI)
    STH=SINF(THETA(J))
    ALT(J)=R

C
C      GEOCENTRIC TO GEODETTIC (TRAJECTORY POINTS)
124 S1=STH
    S2=SINF(2.0*THETA(J))
    S3=SINF(4.0*THETA(J))
    S4=F*S2+F*S3*(REE/R-0.25)
    S5=1.0-F*S1*S1-0.5*F*F*S2*S2*(REE/R-0.25)
    PRLAT=THETA(J)+ASINF(REE*S4/R)
    PRALT=R-REE*S5
    PRLAT=PRLAT/RAD
    PRLON=PHI(J)/RAD
    PRTIME(J)=TIME+TBO-TOPP+.01
    XIM=COSF(THETA(J))*COSF(PHI(J))
    YIM=COSF(THETA(J))*SINF(PHI(J))
    ZIM=SINF(THETA(J))
    ARA=(RLAU/6076.1033+REE*S5)/2.
    ZIZ=XLAU*XIM+YLAU*YIM+ZLAU*ZIM
    RANGE=ARA*ACOSF(ZIZ)
    IF(KBATT)1242,1242,1241
1241 WRITE TAPE KBATT,(FX(I),I=1,3),PRTIME(J),PRLAT,PRLON,PRALT,VELOC,
    1FPA,R
1242 SEC=SECF(PRTIME(J))
    FMIN=FMINF(PRTIME(J))
    HOUR=HOURF(PRTIME(J))
    PN=PN+1.0
    IF(MODF(PN,22)) 126,125,126
125 WRITE OUTPUT TAPE NAT,200,(NAME(I),I=1,9)
    WRITE OUTPUT TAPE NAT,205
126 WRITE OUTPUT TAPE NAT,206,HOUR,FMIN,SEC,PRLAT,PRLON,VELOC,
    1FPA,RANGE
    IF(TIME-TIMP)1261,127,127
1261 GO TO(113,114,116),MM
C
C      LOOK ANGLE
127 DO 135 IT=1,NOST
    STH = XSTH(IT)*RAD

```

```

C
C
SPH = XSPH(IT)*RAD
SHT = XSHT(IT)/6076.1033

      GEODETIC TO GEOCENTRIC (STATION)
XX1=SINF(SHT)
XX2=COSF(SHT)
C=REE/SQRTF(1.0-E*E*XX1*XX1)
S=C-C*E*E
XX3=S+SHT
XX4=C+SHT
STHC=ATANF(XX1/XX2*XX3/XX4)
RS=SQRTF(XX1*XX1*XX3*XX3+XX2*XX2*XX4*XX4)
PN=-1.0
K=1
131 IF(PRTIME(K)-TBO+TOPP-TIMP)1311,135,135
1311 DPHI=PHI(K)-SPH
      ST=SINF(THETA(K))
      CT=COSF(THETA(K))
      X=ALT(K)*CT*COSF(DPHI)-RS*COSF(STHC)
      Y=ALT(K)*CT*SINF(DPHI)
      Z=ALT(K)*ST-RS*SINF(STHC)
      XP=X*XX1-Z*XX2
      ZP=X*XX2+Z*XX1
      ELEV=ATANF(ZP/SQRTF(XP*XP+Y*Y))
      AZIM=ATANF(-Y/XP)
      IF(XP)132,132,130
130  AZIM=PI+AZIM
132  IF(AZIM)1321,1322,1322
1321  AZIM=AZIM+2.*PI
1322  XAZ=AZIM/RAD
      XEL=ELEV/RAD
      SEC=SECF(PRTIME(K))
      FMIN=FMINF(PRTIME(K))
      HOUR=HOURF(PRTIME(K))
      RE=REE/SQRTF(1.+OP*ST*ST)
      PRALT=ALT(K)-RE
      PRLAT=THETA(K)/RAD
      PRLON=PHI(K)/RAD
      SLR=SQRTF(X*X+Y*Y+Z*Z)
      SP6=SPH+WE*(PRTIME(K)-PRTIME(1))
      XX=X*COSF(SP6)-Y*SINF(SP6)
      YY=X*SINF(SP6)+Y*COSF(SP6)

```

```

ANT=ACOSF((XX*AXDOT+YY*AYDOT+Z*AZDOT)/((SQRTF(X*X+Y*Y+Z*Z)*SQRTF(AX
IDOT*AXDOT+AYDOT*AYDOT+AZDOT*AZDOT)))/RAD
PN=PN+1.0
IF(MODF(PN,22)) 134,133,134
133 WRITE OUTPUT TAPE NAT,200,(NAME(I),I=1,9)
WRITE OUTPUT TAPE NAT,209,(TAG(I,IT),I=1,3),XSTH(IT),XSPH(IT),XSH
1(IT)
134 WRITE OUTPUT TAPE NAT,210,HOUR,FMIN,SEC,PRLAT,PRLON,
1XAZ,XEL,SLR,ANT
K=K+1
GO TO 131
135 CONTINUE
190 IF(KBATT)192,192,191
191 END FILE KBATT
192 RETURN
200 FORMAT(1H19A8)
201 FORMAT(39H0
ORBITAL ELEMENTS
1R AXISE28.8,5H N.M.//17H SEMIMINOR AXISE28.8,5H N.M.//15H ECCE
2NTRICITY30.8//9H PERIODE36.8,5H MIN.//18H APOGEE ALTITUDEE27.
38,5H N.M.//19H PERIGEE ALTITUDEE26.8,5H N.M.//14H INCLINATIONE
431.8,5H DEG.//22H ARGUMENT OF PERIGEEE23.8,5H DEG.//25H LONGIT
SUDE OF ASCENDING/23H NODE AT BURNOUT TIMEE22.8,5H DEG.//23H IN
6ITIAL TRUE ANOMALYE22.8,5H DEG.//28H INITIAL ECCENTRIC ANOMALYE1
77.8,5H DEG.)
202 FORMAT(16HVEHICLE ESCAPES)
205 FORMAT(39H KEPLERIAN TRAJECTORY
1 INERTIAL /87H H M S ALTITUDE(NM) LATITUDE LONGITUDE
2 VEL.(FPS) FLT PATH ANG RANGE(NM)//
206 FORMAT(1H F3.0,2F3.0,E14.5,2F12.5,F12.2,F13.5,F13.2)
209 FORMAT(13H0 STATION - 3A8/26XF12.3 6H N.LATF12.3,7H E.LONGF12.0,4
1H FT.//20X23HTRAJECTORY (GEOCENTRIC)13X11HLOOK ANGLES7X5HSLANT5X6H
2ASPECT/6X4HTIME4X8HALTITUDE4X8HLATITUDE3X9HLONGITUDE7X7HAZIMUTH2X9
3HELEVATION3X5HRANGE5X5SHANGLE/4X18HH M S N.MILES6X3HDEG9X3HDEG
412X3HDEG7X3HDEG8X2HNM7X3HDEG)
210 FORMAT(F5.0,2F3.0,F10.0,2F12.2,5X2F10.3,F9.0,F10.2)
SQRTF EQU 76608
END

```

```

C
C
C
C
SUBROUTINE IMPACT(JJ)
  THIS SUBROUTINE COMPUTES THE TRAJECTORY OF THE EMPTY STAGES

      ** * DIMENSION BLOCK * * *

  DIMENSION XX(3),XXD(3),CDM(10,10),XMACH(10,10),AL(26),CBLACK(150),
1NAME(10),OUT(8,42),WD(3),DRA(10),XMN(10),NN(10),DELPR(10),VD(3)
2ACODES(6),DUMB1(3,3),AC(5),AD(5),AG(3,50),AH(50),AK(50),AI(50)
3XIMPA(3,10),XDIMP(3,10),TFIMP(10),RANGE(10)

      ** * COMMON BLOCK * * *

  COMMON DUMB1,NAME,KAA,KAZ,KAB,AC,AD,KAF,AG,AH,AK,AI,Z2,AM,AN,AO,
1AP,XMACH,CDM,NN,NOT,RANGE,XIMPA,XDIMP,TFIMP,DELPR

      ** * EQUIVALENCE BLOCK * * *

  EQUIVALENCE (AL(1),ICODE),{AL(2),ACODE},{AL(3),TRERR},{AL(4),SMIN}
1,{AL(5),SMAX},{AL(6),NOE},{AL(7),SS},{AL(8),T},{AL(9),XD},{AL(10),
2YD},{AL(11),ZD},{AL(12),X},{AL(13),Y},{AL(14),Z},{AL(15),XDD},{AL(
316),YDD},{AL(17),ZDD},{AL(18),XD1},{AL(19),YD1},{AL(20),ZD1},{AL(2
41),ACODES}

      ** * FORMAT BLOCK * * *

  FORMAT(12)
  FORMAT(7F10.0)
  FORMAT(21H1 INTEGRATION ERROR 016)
  FORMAT(19H0 CRT FAILURE AT F9.4,7H SEC***)
  FORMAT(2H1 10A8)
  FORMAT(3X4HTIME4X8HLATITUDE4X9HLONGITUDE4X8HALTITUDE4X5HRANGE/4X3H
1SEC7X3HDEG9X3HDEG9X2HNM9X2HNM/)
  FORMAT(F8.2,2F12.4,F12.2,F10.2)
  FORMAT(3X4HTIME4X8HVELOCITY6X3HFFPA6X7HAZIMUTH/4X3HSEC6X3HFFPS9X3HDE
1G8X3HDEG/)
  FORMAT(F8.2,F11.1,F10.2,F12.2)
  FORMAT(6A8,6HSTAGE=12,2F12.6)
  FORMAT(10X7HSTAGE 12)
      ** * CONSTANTS * * *

  J6=Z2
  T=TFIMP(JJ)
  X=XIMPA(1,JJ)
  Y=XIMPA(2,JJ)
  Z=XIMPA(3,JJ)

```

```

XD=XDIMP{1,JJ}
YD=XDIMP{2,JJ}
ZD=XDIMP{3,JJ}
XX{1}=X
XX{2}=Y
XX{3}=Z
XXD{1}=XD
XXD{2}=YD
XXD{3}=ZD
JP=1
ICODE=3
TRERR=1.E-3
ACODE=1.0
ACODES{1}=1.
ACODES{2}=1.
ACODES{3}=1.
ACODES{4}=1.
ACODES{5}=1.
ACODES{6}=1.
DT=1.
SMIN=.01
SMAX=1.
NOE=6
SS=1./16.
H=T-INTF(T)
H=1.-H
IF(H) 50,51,50
PRTIM1=T+H
GO TO 52
PRTIM1=T+DELPR(JJ)
AE=20925647.
W=7.292115E-5
W2=W*W
XK2=AE*AE*.00162342
GM=1.4076427E16
PI=3.1415927
RAD=PI/180.
PE=.00673852
VEL=SQRTF(XD*XD+YD*YD+ZD*ZD)
R2=X*X+Y*Y+Z*Z
R=SQRTF(R2)
THET=ATANF(Z/SQRTF(X*X+Y*Y))

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50

51

52

```

SIT=SINF(THET)
ALT=R-AE/SQRTF(1.+PE*SIT*SIT)
N=NN(JJ)
DO 30 I=1,N
K=N-I+1
DRA(K)=CDM(I,JJ)
XMN(K)=XMACH(I,JJ)
30
C
C
C
(999)
RTJ      TEXT1
RTJ      ADAMS
ZR0      EQN1
ZR0      TEXT1
ZR0      PRIM1
ZR0      AL
ZR0      CBLACK
ZR0      EXIT1
ZR0      TMIN1
STA      ERR
PRINT 3,ERR
C
EQN1
SLJ
XD1=XD
YD1=YD
ZD1=ZD
VEL=SQRTF(XD*XD+YD*YD+ZD*ZD)
R2=X*X+Y*Y+Z*Z
R=SQRTF(R2)
ARG=(1.+3.*XK2/R2-15.*XK2*Z*Z/R2/R2)/R2/R
GX=-GM*ARG*X
GY=-GM*ARG*Y
GZ=-GM*(ARG+6.*XK2/R2/R2/R)*Z
THET=ATANF(Z/SQRTF(X*X+Y*Y))
SIT=SINF(THET)
ALT=R-AE/SQRTF(1.+PE*SIT*SIT)
IF(ALT) 79,81,81
JP=JP-1
GO TO 4031
81
(82) IF(ALT-500000.) 82,82,83
      LDA      ALT
      RTJ      ATMOS
79

```

```

TEMP      OCT
PRES      OCT
RHO       OCT
VA        OCT
NOP
NOP
85  FMN=VEL/VA
    DRAG = INTERPF(FMN,N,2,XMN,DRA)
    DM=.5*RHO*VEL*VEL*DRAG
    GO TO 84
83  DM=0.
84  XDD=-DM*XD/VEL+GX+2.*W*YD+W2*X
    YDD=-DM*YD/VEL+GY-2.*W*XD+W2*Y
    ZDD=-DM*ZD/VEL+GZ
    GO TO EQN1
EXIT1    SLJ    **
502     CHECK=X
        IF(SENSE LIGHT 2) 999, EXIT1
TMIN1    SLJ    **
        PRINT 4,T
        IF(SENSE SWITCH 5)550,TMIN1
550     SENSE LIGHT 2
        GO TO TMIN1
TEX1     SLJ    **
(3999)   RTJ    EQN1
C        COMPUT THE OUTPUT BLOCK
4000    OUT(1,JP)=T
        CALL GEODED(THET,R,XLAT,XALT)
        OUT(2,JP)=XLAT/RAD
        OUT(4,JP)=XALT/6076.1033
        OUT(3,JP)=ATANF(Y/X)/RAD
        IF(X) 6000,6000,6001
6000    OUT(3,JP)=OUT(3,JP)+180.
6001    OUT(6,JP)=VEL
        VD(1)=XD
        VD(2)=YD
        VD(3)=ZD
        AN1=OUT(3,JP)*RAD+PI/2.
        AN2=PI/2.-XLAT
        CALL ROTATE(AN1,AN2,0.,VD,WD)
        OUT(7,JP)=ATANF(WD(3)/SQRTF(WD(1)*WD(1)+WD(2)*WD(2)))/RAD

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OUT(8,JP)=ATANF(WD(1)/WD(2))/RAD
IF(WD(2)) 6002,6002,6003
6002 OUT(8,JP)=OUT(8,JP)+180.
6003 COSRA=(X*XX(1)+Y*XX(2)+Z*XX(3))/SQRTF(X*X+Y*Y+Z*Z)/SQRTF(XX(1)*XX(
11)+XX(2)*XX(2)+XX(3)*XX(3))
OUT(5,JP) = RANGE(JJ) + 3440.*ACOSF(COSRA)
IF (JP-J6) 4030,4031,4031
4030 IF(ALT)4031,4059,4059
C WRITE OUTPUT BLOCK
4031 WRITE OUTPUT TAPE NOT,5,(NAME(I),I=1,10)
WRITE OUTPUT TAPE NOT, 11,JJ
WRITE OUTPUT TAPE NOT,6
WRITE OUTPUT TAPE NOT,7,((OUT(1,11),I=1,5),I=1,10)
WRITE OUTPUT TAPE NOT,5,(NAME(I),I=1,10)
WRITE OUTPUT TAPE NOT, 11,JJ
WRITE OUTPUT TAPE NOT,8
WRITE OUTPUT TAPE NOT,9,(OUT(1,11),(OUT(1,11),I=6,8),I=1,JP)
IF(ALT) 4037,4061,4061
4061 JP=1
GO TO 4060
C STORE IMPACT POINT FOR PUNCH AND RETURN
4037 PUNCH 10,(NAME(I),I=1,6),JJ,OUT(2,JP), OUT(3,JP)
RETURN
4059 JP=JP+1
4060 PRIMI=PRIMI + DELPR(JJ)
4064 GO TO TEX1
ADAMS EQU 6000B
ATMOS EQU 7300B
SQRTF EQU 7660B
RDF LIB RDF
END
C
C

```

```

SUBROUTINE GEODED(A,B,C,D)
  THIS SUBROUTINE CONVERTS THE GEOCENTRIC COORDINATES TO
  GEODETTIC COORDINATES
  S1=SINF(A)
  S2=SINF(2.*A)
  S3=SINF(4.*A)
  S4=20925647./B-.25
  S5=S2/298.3+S3/88982.89*S4
  S6=1.-S1*S1/298.3-S2*S2*S4/177965.78
  C=A+ASINF(20925647.*S5/B)
  D=B-20925647.*S6
  RETURN
END

```

C  
C

```

SUBROUTINE ROTATE (PHI, THETA, PSI, U, V)
THIS SUBROUTINE GIVES A ROTATION OF CARTESIAN COORDINATES
DIMENSION U(3), V(3), A(3,3), BLOCK(700)
COMMON A, BLOCK
T1=SINF (PHI)
T2=SINF (THETA)
T3=SINF (PSI)
T4=COSF (PHI)
T5=COSF (THETA)
T6=COSF (PSI)
A(1,1)=T6*T4-T5*T1*T3
A(1,2)=T6*T1+T5*T4*T3
A(1,3)=T3*T2
A(2,1)=-T3*T4-T5*T1*T6
A(2,2)=-T3*T1+T5*T4*T6
A(2,3)=T6*T2
A(3,1)=T2*T1
A(3,2)=-T2*T4
A(3,3)=T5
DO 2 I=1,3
V(I)=0.0
DO 2 J=1,3
V(I)=A(I,J)*U(J)+V(I)
RETURN
END
100508

```

1

2

5

ADAMS	ORG	60008		NUMERICAL INTEGRATION
	REM	SOL OF DIFF EQ		RUNGE KUTTA STARTER FOR
	SLJ	0		ADAMS MOULTON DIFF EQ
	SIU	1 AD+13		EVALUATOR
	SIL	2 AD+13		BETA+1 IN INDEX 1
	LIU	1 ADAMS		C(BETA+2) INAC
	LDA	1 1		
	ENQ	0		COMMON TO AD+10
	STQ	AD+10		
	SAL	AD+10		
	ARS	24		
	STQ	AD+9		
	SAL	AD+9		
	LDA	AD+9		
	ENI	2 1		
	INA	1		
AD50	STA	2 AD+13		
+	ISK	2 7		
	SLJ	AD50		
	LDA	7 AD+18		
	AJP	N AD51		
ADERR	LIU	1 AD+13		
	LIL	2 AD+13		
	LDA	ADTHR		
	RAD	ADAMS		
	SLJ	ADAMS		
ADTHR	OCT	300000000		
AD51	AJP	P AD52		
AD53	ENA	0		
	SLJ	ADERR		
ADNMAX	DEC	200		
AD52	STA	AD+12		
	INA	-1		
	STA	AD+21		
	SUB	ADNMAX		
	AJP	P AD53		
	LDA	7 AD+9		
	STA	AD+6		
	AJP	P AD55		
AD56	ENA	2		
	SLJ	ADERR		
AD55	INA	-4		

AJP	P AD56	IF CODE MORE THAN 3 ERROR
LDQ	7 AD+19	DT
ENA	1	AC IS +1
QJP	M ADERR	ERROR DT MINUS
QLS	12	
QJP	P ADERR	ERROR IF DT NOT FLOATING NUM
LDA	1 0	C(BETA+1) TO AC
SAL	ADT1	T TO LOWER ADDRESS AD+1
ARS	24	
SAU	ADEXT	TEXT ADDRESS
SAU	ADEXT1	
SAL	ADEXT2	C(BETA+3) IN AC
LDA	1 2	
SAU	ADTMC	
SAU	ADTMIN	
ARS	24	
SAU	ADEXIT	EXIT ADDRESS
LDA	1 -1	C(BETA) IN AC
SAU	ADY	DERIV ADDRESS
SAL	ADY1	
SAU	ADY3	
SAU	ADY4	
SAU	ADY5	
SAU	ADY6	
SCL	ADMSP	
STA	AD+5	
LDA	AD+20	DATA+7 ADDRESS TK
SAU	ADTS	
SAU	ADTXS	
SAU	ADXDP	
SAL	ADTXR	
INA	1	
SAL	ADXR	DATA+8 ADDRESS YK
SAU	ADXR4	
SAU	ADXR5	
SAL	ADXR3	
SAL	ADXC	
SAU	ADXP	
SAU	ADXMK	
SAU	ADXH	
SAL	ADXP3	
SAU	ADXP4	



SAU	ADDT	ADDRESS DTK
SAU	ADDT1	
SAU	ADDT2	
SAL	ADTC	
SAU	ADDT3	
SAL	ADDT3	COMMON+3N+3
INA	1	ADDRESS DXK
SAU	ADDX	
SAL	ADDX1	
SAL	ADDX2	
SAU	ADDX2	COMMON+UN+3
ADD	AD+12	ADDRESS K2
SAL	ADK2	
SAU	ADK22	
ADD	AD+12	COMMON+5N+3
SAL	ADK3	ADLRESS K3
SAL	ADK22	
ADD	AD+12	COMMON+6N+3
SAL	ADXS	ADDRESS TK-1 OR YAM-7
SAU	ADTIRK	ADDR+7
SAU	ADXS1	
STA	ADDR+7	
INA	1	COMMON+6N+4
SAU	ADXXS	ADDRESS XK-1
ADD	AD+12	COMMON+7N+4
SAL	ADDT	ADDRESS SAVE DT,DX
SAL	ADDT1	AND YAM-6
STA	ADDR+6	
ADD	AD+12	
INA	1	COMMON+8N+5
STA	ADDR+5	ADDRESS YAM-5
ADD	AD+12	COMMON+9N+5
STA	ADDR+4	ADDRESS YAM-4
SAL	ADYRH	
ADD	AD+12	COMMON+10N+5
STA	ADDR+3	ADDRESS YAM-3
ADD	AD+12	COMMON+11N+5
STA	ADDR+2	ADDRESS YAM-2
SAU	ADYRH	
ADD	AD+12	COMMON+12N+5
STA	ADDR+1	ADDRESS YAM-1
STA	AD+12	COMMON+13N+5
ADD		

ADTC2	STA	ADDR	ADDRESS YAM
	LIL	1 AD+12	N TO INDEX 1
	ENA	0	
	STA	1 0	
	IJP	1 ADTC2	
	ENA	6	
	STA	AD+1	RKH TO 6
	STA	AD+3	AMH TO 6
	ENA	0	
	STA	AD	RKC TO 0
	STA	AD+2	AMC TO 0
	STA	AD+4	RKC TO 0
	LDA	AD+5	
	AJP	Z AD17	
	SLJ	AD17	
ADF1	DEC	1	NO PRINT IF P IS ZERO
ADMSP	OCT	77777777777077777	CARD LOU ADDED FOR NO PRINTING
ADDP	LIL	1 AD+12	SEE IF A +
	ENA	0	CLEAR ALL BUT P IN BETA
	STA	AD+4	N TO INDEX 1
	LDA	1 0	CLEAR RKD
ADTC	FAD	1 0	BEGIN DP LOOP
ADTDI	LDQ	1 0	DT + TK LOWER IS B IN TEMP+1
	STQ	ADTEMP	TK UPPER IS C IN ADTEMP
	STA	ADTEMP+1	STORE B
	ENI	2 0	
	QJP	P AD70	
	ENI	2 1	
AD70	FAD	ADTEMP	TK UPPER IS - SET INDEX2,1
	AJP	P AD74	TK UPPER IS + SET INDEX2,0
	INI	2 -1	
AD74	STA	ADTEMP+2	TK UPPER + DT + TK LOWER
	ENQ	0	DO SINGLE PRECISION IF SIGN ALT
	IJP	2 ADXDP	JUMP IF SIGN ALT
	LDA	ADTEMP	C
	AJP	P AD63	
	ENQ	1	
	SCM	ADA7	ABS VALUE C
AD63	STQ	ADSC	STORE SIGN C
	ENQ	0	
	LLS	12	POWER TO MQ
	ARS	3	



SCL	ADMS	EXP TO AC
STA	ADCC2	
LDL	AD3777	JUMP+EXP
THS	AD1777	EXTEND SIGN - EXP
SLJ	AD64	STORE + EXP
SCM	ADNB	
STA	ADCE	B IN AC
LDA	ADTEMP+1	
ENQ	0	
AJP	AD60	1 TO MQ
ENQ	1	ABS VALUE B
SCM	ADA7	STORE SIGN B
STQ	ADSB	
ENQ	0	
LLS	12	
ARS	3	
SCL	ADMS	CLEAR LEAD 3 BITS
STA	ADB	STORE CHAR P
LDL	AD3777	
THS	AD1777	
SLJ	AD61	
SCM	ADNB	ENTEND SIGN NEG POWER
STA	ADBE	STORE POWER B
SUB	ADCE	POWER B - POWER C
AJP	AD71	IF NEG POWERS IS GREATER
LDA	ADTEMP+2	IF C SAME, OR LESS
ENQ	0	
SLJ	ADXDP	CLEAR LESSER
INA	-2000B	
SLJ	AD62	REMOVE BIAS EXP B
INA	-2000B	
SLJ	AD65	REMOVE BIAS EXPC
SCM	ADA7	EXP (-EXP B, ALSO EXP C LARGER
SAU	AD69	SHIFT IN ADDRESS
THS	AD72D	
SLJ	AD67	POWER DIFF GREATER THAN, SKIP
ENQ	0	
LDA	ADB	CHAR B
LRS	0	
STA	ADB	UPPER BITS SCALED B
STQ	ADL	LOWER BITS SCALED B
LDA	ADSB	

AD75	SCM	ADSC	JUMP IF SIGN B DIFF FROM SIGN C
	AJP	N AD66	
	LDA	ADCC2	
	ADD	ADB	ADD CHAR B TO CHAR C
	LDQ	ADL	
	SCQ	2 72	
	LRS	11	
	ALS	11	36 BIT CHAR B + C IN AC
	STA	ADH	LEAD 36 BITS B+C
	QRS	1	0.XXX 1 LEAD BIT MQ
	LDL	ADMP	CLEAR LEAD BIT
	STA	ADHL	LOWER BITS OF B+C
	INI	2 -70	DIFF EXP OF C AND B+C IN 2
	LDA	ADCE	EXP LARGEST OPERAND
	INA	2 0	
	STA	ADHE	EXP OF C+B
	LDQ	ADSC	SIGN C SAME AS SIGN C+B
	QLS	1	
	QJP	Z AD68	
	ENQ	77777B	
	STQ	ADSC	IF SIGN C NEG
	AJP	M AD72	SET ADCE TO -0 FOR SCM
	INA	2000B	JUMP EXP C-
	STA	ADTM	
	LDA	ADHE	+ EXP +BIAS, UPPER PART
	INA	-44B	
	AJP	M AD73	EXP LOWER
	INA	2000B	LOWER EXP- JUMP
	LDQ	ADHL	EXP+ BIAS, LEAST SIGN PART
	LLS	36	LOWER POWER
	SCM	ADSC	LOWER CHAR
	STA	ADTM+1	
	LDA	ADTM	FLOATED LOWER
	LDQ	ADH	UPPER EXP
	QLS	1	UPPER CHAR
	LLS	36	
	SCM	ADSC	UPPER CHAR
	LDQ	ADTM+1	FLOATED UPPER
	SLJ	ADXP	
	INA	-6000B	EXP - UPPER B+C
	STA	ADTM	SET BIAS, CLEAR SIGN BIT OF
	LDA	ADHE	EXP AND NUMBER TO GET BIAS
AD76			
AD68			
AD72			

AD73	INA	-448	EXP LOWER PART B+C
ADXDP	INA	-6000B	SET BIAS
ADTC1	SLJ	AD76	
	STA	1 0	UPPER PART IN DATA
	STQ	1 0	LOWER PART IN COMMON
	IJP	1 ADTC	END DOUBLE PRE LOOP
	SLJ	AD17	
ADMS	OCT	7000000000000000	USE OT CLEAR LEAD 3 BIT OF CHAR
ADNB	OCT	777777777776000	USE TO EXTEND SIGN - POWER
ADA7	OCT	777777777777777	USE TO ALT SIGN
ADMP	OCT	3777777777777777	MAX POS NUMBER
ADTEMP	BSS	3	FLOATED C,B, C+B
AD3777	OCT	3777	LOGICAL MASK EXP
AD1777	OCT	1777	CHECK SIGN EXP
AD72D	DEC	72	
ADSB	BSS	1	SIGN B
ADB	BSS	1	CHAR B
ADBE	BSS	1	EXP B
ADSC	BSS	1	SIGN C
ADCC2	BSS	1	
ADCE	BSS	1	EXP C
ADL	BSS	1	LOWER PART CHAR B
ADH	BSS	1	UPPER CHAR B+C
ADHL	BSS	1	LOWER CHAR B+C
ADHE	BSS	1	EXP B+C
ADTM	BSS	2	
AD66	LDA	ADCC2	
	SUB	ADB	
	INA	-1	
	LQC	ADL	
	SLJ	AD75	
AD17	LIU	1 AD+13	CALC YK
ADY	LIL	2 AD+13	TO EXIT
ADEXIT	SLJ	4 0	
+	SLJ	4 0	
	SIU	1 AD+13	
	SIL	2 AD+13	
	LIL	1 AD+12	
ADTS	LDA	1 0	H TO INDEX 1
	STA	1 0	SAVE DATA TK, YK, IN COMMON
	IJP	1 ADTS	
	LDA	AD+2	AMC TO AC

AD19	AJP	N AD18	VALUE CODE IN AC
	LDA	AD+6	JUMP CODE NOT ZERO
	AJP	N AD19	CODE IS ZERO
	LDA	ADDR+7	ADDRESS YK IS ADDR+7
	SLJ	AD20	CODE NOT ZERO
	INA	-1	JUMP CODE IS 1
	AJP	Z AD21	
	INA	-1	
	AJP	N AD22	JUMP CODE IS 3
	ENA	ADDR+3	CODE IS 2
	SUB	AD	
	SAU	AD23	ADDR+3-RKC
AD23	LDA	0	C(ADDR+3-RKC) IS ADDRESS YK
AD20	SAL	ADYR	SET UP ADDRESS RK STEP
	SAU	ADYR1	
	SAL	ADYK	SAVE YK ADDRESS
	ENA	0	
	STA	AD+4	
	LIL	1 AD+21	CLEAR RKC
	LDA	1 0	N-1 TO INDEX 1
ADYK	STA	1 0	SAVE YK, AMC=0, RKC IS ZERO OR 2
	IJP	1 ADYK	TO INTERRUPT SUBROUTINE
+	SLJ	4 ADT1	EXIT, RK STEP DONE
+	SLJ	4 AD27	
+	SLJ	4 ADRK	C(CODE)
	LDA	AD+6	TO NEXT STEP IF CODE IS ZERO
	AJP	Z AD2P	CODE IS 2
	LDA	AD	RKC-2
	INA	-2	
	AJP	Z AD24	RKC+1 TO RKC
	RAO	AD	
AD24	SLJ	ADDP	RKC IS 2
	ENA	1	SET AMC TO 1, NEXT STEP IS AM
	STA	AD+2	
	SLJ	ADDP	
AD21	LDA	ADDR+5	CODE IS 1
	SLJ	AD25	ADDRESS YK IS ADDR+6
AD22	ENA	ADDR+4	CODE IS 3
	SUB	AD	
	SAU	AD26	
AD26	LDA	0	
	SLJ	AD25	





AJP	M AD34	IF NEG NO DOUBLE
LDA	7 AD+19	
FAD	7 AD+19	
STA	AD+45	
FSB	7 AD+17	LDT
AJP	M AD35	2DT-DT(MAX)
AJP	N AD34	SKIP DOUBLE IF 2DT TO LARGE
LDA	AD+45	DOUBLE DT
STA	7 AD+19	2DT TO DT
ENA	0	
STA	AD	RKC TO ZERO
SLJ	ADDP	
LDA	7 AD+16	CONV TEST FAILED
FSB	7 AD+19	-DT +DT(MIN)
AJP	M AD36	SEE LAST PAGE
LIU	1 AD+13	DT EQUAL OR LESS MIN(DT)
LIL	2 AD+13	
SLJ	4	
SIL	2 AD+13	
SIU	1 AD+13	
ENA	1	
STA	AD+28	DOUBLE TAG SET NON ZERO
ENA	0	
STA	AD+1	RKH TO ZERO
SLJ	AD37	
ENA	0	
LDQ	AD	
STQ	AD+7	
STA	AD+4	
STA	AD	
STA	AD+1	
LDA	AD+6	
INA	-1	
AJP	AD38	
LDA	AD+7	
INA	-1	
AJP	AD38	
LIL	1 AD+21	
LDA	1 0	
STA	1 0	
IJP	1 ADYRH	
ENI	1 0	
		N-1 TO INDEX 1
		YK IN ADDR+2 MOVED TO ADD+4
		MOVE YK FROM RKC 2 TO 0
		CLEAR INDEX 1

ADXP5	LDA	7	AD+19	.5DT TO DT (COMMON)
	FMU		ADC+10	INDEX 1 IS ZERO
	STA	7	AD+19	RESTORE TK-1 TO TK
	LDA	7	ADXS	N-1 TO INDEX 1
	STA	7	ADTS	
	LIL	1	AD+21	
	LDA	1	0	XK+.5 TO XP IF DT AALF
	STA	1	0	
ADXS5	LDA	1	0	XK RESTORED IF DT AALF
	STA	1	0	
	IJP	1	ADXPS	
	LDA	1	AD+6	
	INA		-1	
	AJP	2	AD399	
	LDA		ADDR+4	
AD388	SAL		ADYR	CODE 3
	SAU		ADYR1	REDO RK STEP, HALF DT
	SLJ		AD39	CODE 1 ADDRESS TK
AD399	LDA		ADDR+5	
	SLJ		AD388	AMC NOT ZERO
	LDA		ADDR	BEGIN AM STEP
AD18	SAL		ADYAS	N-1 TO INDEX 1
	LIL	1	AD+21	
	LDA	1	0	SAVE YK IN ADDR
ADYAS	STA	1	0	
	IJP	1	ADYAS	
	ENA		0	
ADMSET	STA		AD+4	CLEAR RKD
	LDA		ADDR	ADDRESS SETUP AM STEP
	SAL		ADYR	YAM SET FOR RK STEP
	SAU		ADYR1	
	SAU		ADYMP	AM STEP
	SAL		ADYMC	YAM-1
	LDA		ADDR+1	
	SAU		ADYMP1	
	SAL		ADYMC1	YAM-2
	LDA		ADDR+2	
	SAU		ADYMP2	
	SAU		ADYMC2	
	LDA		ADDR+3	
	SAL		ADYMP3	YAM-3
	SLJ	4	ADTI	GO TO INTERRUPT SUBROUTINE



+ + +

AD40  
+

ADTMC  
+

AD43

ENI  
SLJ  
LDA  
INA  
AJP  
LDA  
LDQ  
STA  
LDA  
STQ  
LDQ  
STA  
STQ  
SLJ  
SLJ  
AJP  
ENA  
STA  
ENA  
STA  
LDA  
FSB  
AJP  
LIU  
LIL  
SLJ  
SIU  
SIL  
SLJ  
LDA  
AJP  
LDA  
INA  
AJP  
LDA  
SUB  
AJP  
LDA  
FAD  
STA  
FSB  
AJP

0  
4 ADAMM  
AD+6  
-2  
N AD40  
AJP  
LDA  
LDQ  
STA  
LDA  
STQ  
LDQ  
STA  
STQ  
SLJ  
SLJ  
AJP  
ENA  
STA  
ENA  
STA  
LDA  
FSB  
AJP  
LIU  
LIL  
SLJ  
SIU  
SIL  
SLJ  
LDA  
AJP  
LDA  
INA  
AJP  
LDA  
SUB  
AJP  
LDA  
FAD  
STA  
FSB  
AJP

EXIT SAME LOGIC EXIT +1  
DO AM STEP  
CODE IN AC

JUMP IF CODE 3  
CODE 2, YAM IN AC  
SHIFT ADDRESS YAM  
ADDRESS YAM TO YAM-1

ADDRESS YAM-1 TO YAM-2

ADDRESS YAM-2 TO YAM-3  
ADDRESS YAM-3 TO YAM  
NEXT STEP

JUMP IF YK+1 CONV  
CONV TEST FAILED  
SET AMH TO 0

SET AMC TO 1

DT(MIN)-DT  
HALF DT IF MINUS  
DT TO SMALL DO NOT HALF DT

ACCEPT DT STEP  
DOUBLE TAG  
IF NO DOUBLE JUMP

NO DOUBLE IF AMC LESS THAN 4

NO DOUBLE IF AMH LESS THAN 6

2DT

2DT-MAX DT  
STEP TO LARGE NO DOUBLE

AD46	LDA	AD+45	DOUBLE DT
	STA	7 AD+19	2DT TO DT
	ENA	1	
	STA	AD+2	AMC TO 1
	LDA	ADDR+3	ADJ ADDRESS YK FOR DOUBLE
	LDQ	ADDR+2	
	STA	ADDR+2	YAM-3 TO YAM-2
	LDA	ADDR+5	
	STA	ADDR+3	YAM-5 TO YAM-3
	STQ	ADDR+5	YAM-2 TO YAM-5
	LDA	ADDR+7	
	LDQ	ADDR+4	
	STA	ADDR+4	YAM-7 TO YAM-4
	STQ	ADDR+7	
	SLJ	ADDP	
AD44	AJP	Z AD46	IF 2DT EQ DT MAX, DOUBLE
AD41	SLJ	AD41	DT STEP ONLY
AD42	RAO	AD+2	BUMP AMC AND AMH BY 1
	RAO	AD+3	SHIFT ADDRESS YAM
	LDQ	ADDR	
	LDA	ADDR+1	YAM TO YAM-1
	STQ	ADDR+1	
	LDQ	ADDR+2	YAM-1 TO YAM-2
	STA	ADDR+2	
	LDA	ADDR+3	YAM-2 TO YAM-3
	STQ	ADDR+3	
	LDQ	ADDR+4	YAM-3 TO YAM-4
	STA	ADDR+4	
	LDA	ADDR+5	
	STQ	ADDR+5	YAM-5 TO YAM-6
	LDQ	ADDR+6	
	STA	ADDR+6	YAM-6 TO YAM-7
	LDA	ADDR+7	YAM-7 TO YAM
	STQ	ADDR+7	
	STA	ADDR	
	SLJ	ADDP	
AD30	ENA	0	RK STEP IN INTERRUPT SECRKSTEP
	STA	AD	CLEAR RKC
ADTIRK	LIL	1 AD+12	
	LDA	1 0	RESTORE OLD TK,XK FROM
	STA	1 0	ADDR+7 TO STANDARD COMMON
	IJP	1 ADTIRK	DUM FIRST DT STEP + INT RK STEP

ADRK	SLJ	ADDP	SUBROUTINE RK STEP
	SLJ	0	
	LDA	7 AD+19	.5DT TO AD+22
	FMU	ADC+10	
	STA	AD+22	
	LIL	1 AD+21	N-1 TO INDEX 1
	FAD	7 ADTK	TK+.5DT TO DATA
	STA	7 AD+20	
ADYR	LDA	AD+22	LOOP TO GET XK+.5DT(YK)
	FMU	1 0	.5DT(YK)
ADXR	FAD	1 0	ADD XK
	STA	1 0	STORE YK+.5
	IJP	1 ADYR	END LOOP TO GET YK+.5DT(YK)
	LIL	2 AD+13	
	LIU	1 AD+13	RESTORE INDEX
ADY4	SLJ	4 0	CALC U(T+.5DT,XK+.5DTYK)
+	SIU	1 AD+13	SAVE INDEX
	LIL	1 AD+21	N-1 TO INDEX 1
ADK2	LDA	1 0	LOOP TO GET X IN ARG K3
	STA	1 0	STORE K2
ADXR1	FMU	AD+22	.5DT(YT+.5DT,XK+.5DTYK)
	FAD	1 0	ADD YK FOR ARG K3
ADXR4	STA	1 0	STORE X, ARG K3
	IJP	1 ADK2	END LOOP TO GET X, ARG K3
	LIU	1 AD+13	
ADY5	SLJ	4	CALC Y IN K3, RK STEP
+	SIU	1 AD+13	
	LDA	7 AD+19	DT TO AD+23
	STA	AD+23	TK+DT TO DATA
	FAD	7 ADTK	
	STA	7 AD+20	DT
	LDA	7 AD+19	DIV DT BY 6
	FDV	ADC+11	
	STA	AD+24	
	LIL	1 AD+21	N-1 TO INDEX 1
ADK3	LDA	1 0	LOOP TO GET X IN K4
	STA	1 0	STORE X3, RK STEP
ADXR2	FMU	AD+23	DT(K3)
	FAD	1 0	XK+K3DT IS ARG X IN K4
ADXR5	STA	1 0	
	IJP	1 ADK3	END LOOP TO GET ARGX, K4
	LIU	1 AD+13	

ADY6	SLJ	4		CALC Y IN K4, RK STEP
+	SIU	1	AD+13	
	SIL	2	AD+13	
	LIL	1	AD+21	N-1 TO INDEX 1
	LDA	1	0	K2, LOOP TO CALC DT
ADK22	FAD	1	0	K2+K3
	STA		AD+27	
	FAD		AD+27	
	FAD	1	0	2K2+2K3
ADYR1	FAD	1	0	+K1
	FAD	1	0	+K4
	FAD		AD+24	.1666DT(K1+2K2+3K3+K4)
	FAD		AD+4	RKD IN MQ
ADDX1	LDQ		AD+4	
	QJP	N	AD3	
	STA	1	0	RKD IS ZERO, STORE DX
	SLJ		AD+27	
AD3	STA		AD+27	RKD NOT ZERO
ADDX2	FAD	1	0	SAVE DX THIS STEP
	STA	1	0	CUMULATE DX
	LDA		AD+27	RESTORE DX THIS STEP
ADXR3	FAD	1	0	ADD XK TO DX
	STA	1	0	STORE YK+1 IN DATA
	IJP	1	ADK22	END LOOP TO CALC DT, IN RK
	LDA	7	AD+19	DT IN AC
	LDQ		AD+4	RKD IN MQ
	QJP	N	ADT3	
	STA	7	ADT3	DT TO DT COMMON IF RRD IS ZERO
	SLJ		ADRK	
ADDT3	FAD	0	0	
	STA	0	0	
	SLJ		ADRK	
ADCV	SLJ	0	0	CONV SUBROUTINE
	ENA	0	0	CLEAR DOUBLE TAG
	STA		AD+28	N-1 TO INDEX 1
	LIL	1	AD+21	SAVE INDEX 3
	SIU	3	AD+33	E
	LDA	7	AD+15	E TO AD+34
	STA		AD+34	DIV E BY DOUBLE FACTOR
	FDV		ADC+2	STORE DOUBLE ERROR
	STA		AD+35	A IN AC
	LDA	7	AD+14	JUMP IF A ZERO
	AJP	Z	ADA	
	AJP	P	AD4	

ADA	SCM	ADMASK	ABS VAL A IN AC
	ENI	2 0	CLEAR INDEX2, AI IS A
	STA	0	
	SLJ	ADA1	
AD4	LIL	2 AD+21	N-1 TO INDEX 2 IF A+
ADA1	LDA	2 0	BEGIN CONV LOOP
	STA	AD+30	A TO AD+30
ADXP2	LDA	1 0	XP
	AJP	P ADXC	
	SCM	ADMASK	ABS VALUE XP
ADXC	STA	AD+29	XP TO AD+29
	LDA	1 0	XC
	ENI	3 2	2 TO INDEX 3
	AJP	P AD5	
AD5	SCM	ADMASK	
	THS	3 AD+29	HALF EXIT A GREATEST
	SLJ	AD6	REPLACE A BY LARGER TABLE VAL
	LDA	3 AD+29	
	ENQ	3 0	
	QJP	N AD5	IF INDEX 3 IS ZERO, END TABLE
AD6	STA	AD+32	STORE LARGEST 4, XP, XC
	AJP	2 AD7	SKIP CONU TEST IF ALL ZERO
ADXP3	LDA	1 0	XP
	FSB	1 0	XP-XC
	FDV	AD+32	DIV BY LARGEST OF A, XP, XC
	AJP	P AD8	
AD8	SCM	ADMASK	
	FSB	AD+35	
	AJP	M AD7	
	AJP	2 AD7	SKIP IF ZERO, CONV DOUBLE
	STA	AD+28	SET DOUBLE TAG NON ZERO, NO DOUBLE
	FSB	AD+34	-E, DOUBLE TEST FAILED
	AJP	P AD10	IF AC+ CONV TEST FAILED
AD7	IJP	2 AD9	
AD9	IJP	1 ADA1	END CONV TEST LOOP
	ENA	0	RESTORE INDEX 3
AD57	LIU	3 AD+33	
	SLJ	ADCV	CONV TEST PADDED
AD10	ENA	-1	
	SLJ	AD57	CONV TEST FAILED
ADMASK	OCT	-1	
ADAMM	SLJ	0	SUBROUTINE AM STEP

ADYMP3	1 LIL	1 AD+21	N-1 TO INDEX 1
	7 LDA	7 AD+19	DT
		ADC+3	DIV DT BY 24, PUT IN AD+25
	STA	AD+25	
	LAC	ADC+4	LOOP TO CALC XP
	FMU	1 0	
	STA	AD+26	- .9YAM-3
	LDA	ADC+5	
ADYMP2	FMU	1 0	37 YAM-2
	FAD	AD+26	
	STA	AD+26	
	LDA	ADC+6	
ADYMP1	FMU	1 0	-59YAM-1
	FAD	AD+26	
	STA	AD+26	
	LDA	ADC+7	
ADYMP	FMU	1 0	55.YAM
ADXM	FAD	AD+26	(55YAM-59YAM-1+37YAM-2-9YAM-3)
	FMU	AD+25	MULT BY DT OVER 24
ADXP	FAD	1 0	ADD XK IN COMMON
	STA	1 0	XP TO DATA
	STA	1 0	YP TO COMMON
	IJP	1 ADYMP3	END LOOP TO CLAC XP
	LDA	7 AD+19	DT IN DATA
ADTK	STA	7 AD+13	DT TO DT COMMON REGION
	FAD	0	TK FROM COMMON
	STA	7 AD+20	TTAT TO T IN DATA
	LIU	1 AD+13	
	LIL	2 AD+13	RESTORE INDEX
	SLJ	4 0	CALC YP IN AM
ADY3	SIU	1 AD+13	SAVE INDEX
+	SIL	2 AD+13	N-1 TO INDEX 1
	LIL	1 AD+21	LOOP TO CALC XC
ADYMC1	LDA	ADC+8	-5 YAM-1
	FML	1 0	YAM-2 -5YAM-1
ADYMC2	FAD	1 0	
	STA	AD+26	
ADYMC	LDA	ADC+9	
	FMU	1 0	19YAM
	FAD	AD+26	
ADYP	STA	AD+26	
	LDA	ADC+4	

ADDX	FMU	1 0	9YP
	FAD	AD+26	MULT BY DT OVER 24
ADXMK	FMU	AD+25	STORE DX
	STA	1 0	ADD XK FROM COMMON
	FAD	1 0	STORE YK+1 IN DATA
	STA	1 0	END LOOP TO CALC XC
ADINT	IJP	1 ADYMC1	EXIT AM STEP SUBROUTINE
	SLJ	ADAMM	INTERPOLATE FOR HALF STEP
	LDA	ADDR	
	SAU	ADI	ADDRESS YAM
	SAL	ADI1	
	LDA	ADDR+1	
	SAU	ADI1	ADDRESS YAM-1
	SAU	ADI11	
	LDA	ADDR+2	
	SAL	ADI2	ADDRESS YAM-2
	SAL	ADI22	
	LDA	ADDR+3	
	SAU	ADI3	ADDRESS YAM-3
	SAU	ADI33	
	SAL	ADI333	
	LDA	ADDR+4	
	SAL	ADI4	ADDRESS YAM-4
	SAL	ADI44	
	LDA	ADDR+5	
	SAU	ADI5	ADDRESS YAM-5
AD14	LIL	1 AD+21	N-1 TO INDEX 1
	LDA	ADC+12	BEGIN INTERPOLATE LOOP
	FMU	1 0	
	STA	AD+36	.0234575YAM-4
	LDA	ADC+13	
AD13	FMU	1 0	-.15625YAM-3
	STA	AD+37	
AD12	LDA	ADC+14	.703125YAM-2
	FMU	1 0	
	STA	AD+38	
	LDA	ADC+15	.46875YAM-1
AD11	FMU	1 0	
	STA	AD+39	
AD144	LDA	ADC+16	
	FMU	1 0	-.0390625YAM-4
	STA	AD+40	

ADI33	LDA	ADC+17	
	FMU	1 0	.21875YAM-3
ADI22	STA	AD+41	
	LDA	ADC+18	
	FMU	1 0	-.546875YAM-2
ADI11	STA	AD+42	
	LDA	ADC+19	
	FMU	1 0	1.09375YAM-1
ADI	STA	AD+43	
	LDA	ADC+20	.2734375YAM
	FMU	1 0	
ADI5	FAD	AD+43	
	FAD	AD+42	
	FAD	AD+41	
	FAD	AD+40	
	STA	1 0	PUT YAM-.5 IN YAM-5
ADI	LDA	ADC+16	
	FMU	1 0	
	FAD	AD+39	
	FAD	AD+38	
	FAD	AD+37	
	FAD	AD+36	
ADI333	STA	1 0	PUT YAM-1.5 IN YAM-3
	IJP	1 AD14	END INTERPOLATE LOOP
	LDA	ADDR+5	RESHUFFLE ADDRESS
	LDQ	ADDR+1	FOR HALF DT
	STA	ADDR+1	ADDRESS YAM-5 TO YAM-1
	LDA	ADDR+2	ADDRESS YAM-1 TO YAM-2
	STQ	ADDR+2	ADDRESS YAM-2 TO YAM-4
	LDQ	ADDR+4	ADDRESS YAM-4 TO YAM-5
	STA	ADDR+4	
	STQ	ADDR+5	
	LDA	AD+19	.5DT TO DT
	FMU	ADC+10	
	STA	7 AD+19	
	LIL	1 AD+12	
ADTXR	LDA	1 0	MOVE TK, YK IN COMMON
	STA	1 0	TO DATA
	IJP	1 ADTXR	
	SLJ	ADMSET	
ADTI	SLJ	0	TIME INTERRUPT SUBROUTINE
	ENA	0	EXIT INTERRUPT IF T IS ZERO



AD11	AJP	Z	AD11	C(T) TO A C
	LDA	7	ADT1	
	AJP	N	ADEXT	IF C(T) ZERO SKIP SUBROUTINE
	LDA		ADONE	10000000 TO AC
	RAD		ADT1	BUMP UPPER ADDRESS BY 1
	SLJ		ADT1	GO TO EXIT+1 NO RK STEP
	ENA		0	
ADEXT	AJP	N	AD12	
	ENA		-1	IF C(T) NOT ZERO BUT TEXTIT ZERO
	SLJ		ADERR	GO TO ERROR RETURN BETA+4, AC-1
AD12	LDA	7	ADT1	
	FSB	7	ADTK	C(T)-TK
	AJP	Z	AD13	JUMP IF CT EQUALS TK
	AJP	M	AD13	
	FSB	7	AD+19	-DT+C(T)-TK, +AND NOT ZERO
	AJP	P	AD11	TO AD11, IF NO INTERRUPT
AD12A	LDA		AD+4	
	STA		AD+31	SAVE RKD
	AJP	Z	AD14	N TO INDEX 1
	LIL	1	AD+12	
	LDA	1	0	SAVE DT, DX IN INTERRUPT
	STA	1	0	IF RKD IS 1
ADDT	IJP	1	ADDT	INTERUPT LOOP
	LDA	7	AD+19	SAVE DT IN DATA
AD14	STA		AD+44	
	ENQ		0	SET RKD TO 0
	STQ		AD+4	
	LDA	7	ADT1	C(T)-TK IS DT
	FSB	7	ADTK	DO RK DE STEP
	STA	7	AD+19	
	SLJ	4	ADRK	RESTORE OLD DT
	LDA	7	AD+44	SET AC 0
	STA		AD+19	
	ENA		0	TK ADVANCED TO C(T), GO TO TEXTIT
	LIL	2	AD+13	
ADEXT2	LIU	1	AD+13	
	SLJ	4	0	
	SIU	1	AD+13	
	SIL	2	AD+13	
	LDA	7	ADT1	
	FSB	7	AD+20	C(T)-TK IN DATA
	AJP	Z	AD15	

AJP	M AD15	IF C(T)-TK IS 0 OR - EXIT INT-KOOP
LDA	7 ADT1	C(T)-TK IN COMMON
FSB	7 ADTK	C(T)-TK-DT
FSB	7 AD+19	EXIT INTERRUPT IF +
AJP	P AD15	LOOP INTERRUPT
SLJ	AD14	
LIU	1 AD+13	IF C(T)-TK ZERO OR -
LIL	2 AD+13	INTERUPT WITHOUT RK STEP
SLJ	4 0	
SIU	1 AD+13	
SIL	2 AD+13	
LDA	7 ADT1	IS NEW PRINT TIEM WITHIN DT OF
FSB	7 ADTK	PRESENT TIME
FSB	7 AD+19	NO, NORMAL EXIT +1 INTERRUPT
AJP	P AD11	YES8 DO RK STEP WITH MODIFIED DT
SLJ	AD12A	
OCT	100000000	
LDA	AD+31	RESTORE RKD
STA	AD+4	SKIP RESTORE IF RKD IS ZERO
AJP	Z AD16	N TO INDEX 1
LIL	1 AD+12	
LDA	1 0	DX+OLD DX TO DX
FAD	1 0	CUMULATE AND RESTORE DT
STA	1 0	ID RKD IS 1
IJP	1 ADDT1	EXIT, RK STEPS DONE
SLJ	ADT1	
DEC	4,6,32.	20. IS DOUBLE FACTOR
DEC	24.,9.,37.,-59.,55.	
DEC	-5.,19.,5,6.	
DEC	.0234375,-.15625	
DEC	.703125,.46875	
DEC	-.0390625,.21875	
DEC	-.546875,1.09375	
DEC	.2734375	
BSS	50	
BSS	9	

AD13	
ADEXT1	
+	
ADONE	
AD15	
ADDT1	
ADDT2	
AD16	
ADC	
AD	
ADDR	



LDA	1	TABLE-1	HB
FSB		COMP	H
FMU		QCONST	
FDV	1	TABLE2X	TMB
SLJ	0	AROUND	EXP ARGUMENT IS IN A REGISTER
NONZERO	1	TABLE2X	TMB
LDA		COMP+1	
FDV		COMP+3	
STA		COMP+3	
ENA		LOGF	
RTJ	0		
QCONST		3.41647942D-02	
+		QCONST	
FDV	1	TABLE1X	EXP ARGUMENT NOW IN A REGISTER
STA		COMP+3	
ENA		COMP+3	
RTJ	0	EXPF	
MTOFEET		3.2808333333	
+	1	TABLE3X	
PRESEXT		***	PB
FDV		COMP+1	PRESSURE
FMU		RHOEXT	TM
STA		***	DENSITY
LDA		COMP+1	TM
FDV		TABLE2X	BASE TEMPERATURE
STA		COMP+3	
ENA		COMP+3	
RTJ	0	SQRTF	
CSZERO		1116.4437	
VSNDEXT		CSZERO	VELOCITY OF SOUND
STA		***	
EXIT2	1	OCTONE	
LIU	0	***	
SLJ		3.2365983D-04	
RHOEXT		OCTONE	
OCTONE		000000000001	
COMP	4		
HFACT		6356766.	
DEC		0.0	
TABLE		0.,11000.,20000.	
DEC		32000.,47000.,52000.	
DEC		61000.,79000.,88743.	
DEC		98451.,108129.,117777.	
DEC		146542.,156071.,165572.	

DEC	184485.,221968.,286478.
DEC	376315.,463530.,548235.
DEC	630536.,700000.
TABLE1X DEC	-.0065,-.0065,0.0
DEC	.001,.0028,0.0,-.002
DEC	-.004,0.0,.00309,.0051663
DEC	.0103648,.0208587,.015741,.0105252,.0074023
DEC	.00533575,.0043404,.0036733,.00298114
DEC	.002007,.00133655,.00133655
TABLE2X DEC	288.15,288.15,216.65
DEC	216.65,228.65,270.65
DEC	270.65,252.65,180.65
DEC	180.65,210.65,260.65
DEC	360.65,960.65,1110.65
DEC	1210.65,1350.65,1550.65
DEC	1830.65,2160.65,2420.65
DEC	2590.65,2700.65
TABLE3X DEC	2116.2169,2116.2169,472.6792
DEC	114.3415,18.12814,2.316195
DEC	1.232178,.380303,.0216707
DEC	3.43294E-3,6.28025E-4,15.356435E-5
DEC	5.26501E-5,10.5678E-6,7.71278E-6
DEC	5.83017E-6,3.51815E-6,14.53042E-7
DEC	3.93231E-7,8.412236E-8,2.2867465E-8
DEC	7.200254E-9,2.48704E-9
BSS	100
EXP	EXP
LOG	LOG
LOGF	LOGF
SQRT	SQRT
SQRTF	SQRTF
LIB	LIB
END	END

TDR-63-11

8. SPURT SAMPLE PRINTOUT DATA.

SPURT SAMPLE PRINTOUT DATA

1. INPUT DATA - Launch Parameters and Spin Table
2. INPUT DATA - Stage Weight and Aerodynamic Parameters --  
1 page per Stage
3. SPURT OUTPUT DATA
4. SPURT OUTPUT DATA
5. SPURT OUTPUT DATA
6. SPURT OUTPUT DATA
7. TWO-BODY OUTPUT DATA - Orbital Elements
8. TWO-BODY OUTPUT DATA - Keplerian Trajectory
9. TWO-BODY OUTPUT DATA - Look-Angles
10. IMPACT OUTPUT DATA - Position Information
11. IMPACT OUTPUT DATA - Velocity Information

INPUT FILE	35 LO PL	NEW DATA JANUARY 6, 1968	40 SEC CONST 50 DBO LNU						
		20.9136	.0001	100.0000	0.10	240.00	0.00	0.00	0.00
	35.0000								

## SPIN TABLE

[illegible]

**Flst Line - Name - 80 Hollerith Characters.**

2nd Line - Payload Weight, Latitude, Longitude, Launch Coordinate System Origin Altitude, Launch Azimuth, and Control Numbers.

### Spin Table Columns

[illegible]



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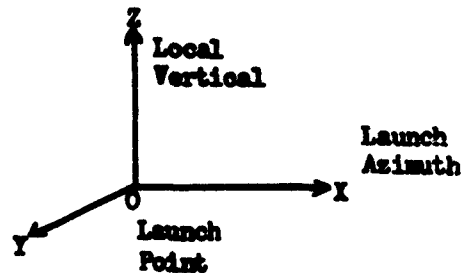
## DEFINITIONS

### DEFINITIONS:

All trajectories are with respect to an oblate earth.

### RANGE COORDINATE SYSTEM:

A left hand orthogonal cartesian coordinate system with the XY plane tangent to an oblate earth at the launch point. The Z axis is positive upward along the local vertical and the X axis is positive along the launch azimuth.



### TIME (Sec):

Relates to the first motion of the vehicle off the launch site.

### LATITUDE (Deg):

Angle between the normal to the reference spheroid passing through the vehicle and the equatorial plane. Positive in the northern hemisphere.

### LONGITUDE (Deg):

Angular distance measured from the foot of the Greenwich meridian to the vehicle sub-point meridian. West of Greenwich is negative and East is positive.

ALTITUDE (Feet):

Distance from vehicle to the surface of a geodetic earth below the vehicle.

VELOCITY (fps):

Velocity of vehicle with respect to a point on a rotating earth directly below the vehicle.

AZIMUTH (Deg):

Angle from North the velocity vector makes with the local meridian. Measured positive C. W. from North.

E. P. A. (Deg):

Flight path angle. The angle the velocity vector makes with the local horizon; positive upward.

RANGE (N.M.):

Arc distance from launch to the point under the vehicle measured along the surface of the earth.

DEFLECTION (N.M.):

Distance the vehicle has deviated from the launch axis in the XY plane.

PHI (Deg):

Euler angle between the longitudinal axis of the vehicle and the Z axis in the range coordinate system

THETA (Deg):

Euler angle between the longitudinal axis of the vehicle and the XZ plane measured in the rotated X'Y' plane.

THRUST (Lbs):

Thrust of the rocket motor with thrust increase due to pressure decrease included.

WEIGHT (Lbs):

Weight of the remaining portion of the vehicle. Given in terms of sea level pounds.

TOTAL ACCEL. ( $g$ 's):

Absolute total acceleration of the vehicle normalized by the gravitational constant  $g_0$ .

DYNAMIC PRESSURE ( $Lb/Ft^2$ ):

The classical dynamic pressure on the vehicle given by  $1/2$  of the density times the velocity squared.

DRAG ( $Lbs$ ):

Aerodynamic drag on the vehicle.

MACH NO:

Mach number of the vehicle.

X, Y, Z ( $Ft$ ):

Position vector of vehicle in range coordinate system

X-DOT, Y-DOT, Z-DOT ( $f\cdot s$ ):

Velocity vector of vehicle in range coordinate system

ORBITAL ELEMENTS:

(Ref 3) The parameters of a classical Keplerian ellipse based on an inverse square gravitational field.

SEMI MAJOR AXIS ( $a$ ):

One half of the longest diameter of the Keplerian ellipse.

SEMI MINOR AXIS ( $b$ ):

One half of the smallest diameter of the Keplerian ellipse.

ECCENTRICITY ( $e$ ):

A measure of the flattening of the Keplerian ellipse.

PERIOD ( $p$ ):

The time that a space vehicle takes to make one complete orbit.

APOGEE ALTITUDE ( $h_A$ ):

The distance to the highest point on the ellipse from the surface of the Earth.

PERIGEE ALTITUDE ( $h_p$ ):

The radius of the point on the ellipse closest to the Earth, minus the radius of the Earth.

INCLINATION (i):

The angle between the orbit plane and the equatorial plane.

ARGUMENT OF PERIGEE ( $\omega$ ):

The angular distance measured in the orbit plane from the line of nodes to the line of apsides.

LONGITUDE OF THE ASCENDING  
NODE AT BURNOUT TIME ( $\Omega$ ):

The angular distance measured at burnout time from Greenwich eastward in the equatorial plane to the point of intersection of the orbit plane where the vehicle crosses from south to north.

INITIAL TRUE ANOMALY ( $V_0$ ):

The angle measured at burnout time at the center of the Earth between the line of apsides and the radius vector to the vehicle measured from perigee in the direction of motion.

INITIAL ECCENTRIC ANOMALY ( $E_0$ ):

The angle at the center of the ellipse between the line of apsides and radius vector of the auxiliary circle through a point which has a projection of the ellipse corresponding to the initial true anomaly.

H. M. S.:

Time in hours, minutes, and seconds of the vehicle from launch.

INERTIAL VELOCITY (fps):

The velocity of the vehicle with respect to a coordinate system fixed to, but not rotating with, the Earth.

**INERTIAL FLIGHT PATH ANGLE**  
**(Deg):**

The angle at any given time the inertial velocity vector makes with respect to the perpendicular to the radius vector at that time.

**LOOK ANGLES:**

The direction to position a tracking antenna at a given station.

**AZIMUTH (Deg):**

The angle that the projection in the horizontal plane of the vector pointing to the vehicle makes with the North direction.

**ELEVATION (Deg):**

The angle from the horizon to the vector pointing to the vehicle.

**SLANT RANGE (N.M.):**

The distance from the tracking station to the vehicle.

**ASPECT ANGLE (Deg):**

Angle between the vehicle spin axis and a vector from a given station to the vehicle.

## 32 LB PAYLOAD NOVEMBER 15, 1962

LOCAL FLIGHT PARAMETERS						
TIME	LATITUDE	LONGITUDE	ALTITUDE	VELOCITY	AZIMUTH	FPA
SEC	DEG	DEG	FEET	FT/SEC	DEG	DEG
.00	28.5136	-80.5765	10		105.00	76.00
1.00	28.5136	-80.5763	50	96	105.16	71.32
2.00	28.5136	-80.5763	190	200	105.15	71.46
3.00	28.5135	-80.5761	428	302	105.08	71.14
4.00	28.5134	-80.5757	760	402	104.88	70.27
5.00	28.5133	-80.5752	1184	504	104.58	69.80
6.00	28.5132	-80.5746	1699	608	104.41	67.15
7.00	28.5130	-80.5738	2305	716	104.58	66.00
8.00	28.5128	-80.5729	3007	827	104.58	65.17
9.00	28.5125	-80.5717	3806	941	104.53	64.17
10.00	28.5122	-80.5704	4702	1057	104.59	63.44
11.00	28.5118	-80.5689	5695	1172	104.58	62.84
12.00	28.5114	-80.5672	6784	1288	104.60	61.98
13.00	28.5110	-80.5652	7970	1407	104.64	61.35
14.00	28.5105	-80.5631	9254	1528	104.63	60.72
15.00	28.5100	-80.5607	10637	1652	104.65	60.17
16.00	28.5094	-80.5581	12122	1780	104.68	59.85
17.00	28.5087	-80.5553	13769	1910	104.68	59.14
18.00	28.5080	-80.5522	15401	2043	104.69	58.67
19.00	28.5072	-80.5489	17200	2180	104.71	58.23
20.00	28.5064	-80.5453	19107	2320	104.72	57.80
21.00	28.5055	-80.5415	21127	2465	104.72	57.39
22.00	28.5045	-80.5373	23262	2616	104.74	57.01
23.00	28.5035	-80.5329	25518	2775	104.76	56.46
24.00	28.5024	-80.5282	27900	2941	104.77	56.31
25.00	28.5012	-80.5231	30414	3117	104.77	55.98
26.00	28.5000	-80.5177	33068	3301	104.79	55.67
27.00	28.4986	-80.5119	35868	3492	104.81	55.38
28.00	28.4972	-80.5057	38813	3676	104.81	55.10
29.00	28.4957	-80.4993	41889	3831	104.82	54.83
30.00	28.4941	-80.4925	45065	3946	104.84	54.57
31.00	28.4925	-80.4856	48308	4020	104.85	54.32
32.00	28.4908	-80.4785	51588	4066	104.86	54.06
33.00	28.4891	-80.4712	54888	4097	104.87	53.81

NOVEMBER 15, 1962

32 LB PAYLOAD

TIME SEC	RANGE N.MILES	DEFLECTION N.MILES	ALTITUDE N.MILES	VELOCITY FT/SEC	PHI DEG.	THETA DEG.
0.00	.000	-.000	.002	.0	14.000	-.000
1.00	.002	-.000	.008	95.7	14.017	-.000
2.00	.010	-.000	.031	200.1	14.313	-.016
3.00	.023	-.000	.070	301.6	15.621	-.114
4.00	.042	-.000	.125	401.9	16.652	-.362
5.00	.068	-.000	.195	503.7	22.575	-.598
6.00	.103	-.000	.280	608.0	24.220	-.358
7.00	.146	.001	.379	715.5	23.001	.096
8.00	.199	.001	.495	826.8	24.978	-.358
9.00	.261	.002	.626	941.0	25.991	-.131
10.00	.333	.002	.774	1056.7	26.609	-.300
11.00	.417	.003	.937	1172.0	26.950	.005
12.00	.511	.004	1.117	1288.3	28.654	-.403
13.00	.616	.004	1.312	1407.1	28.276	-.103
14.00	.731	.005	1.523	1528.3	29.124	-.049
15.00	.860	.006	1.751	1652.4	30.344	-.391
16.00	1.002	.007	1.995	1779.5	30.160	-.143
17.00	1.156	.008	2.256	1909.7	30.546	.029
18.00	1.324	.009	2.535	2042.9	31.679	.292
19.00	1.505	.010	2.831	2179.6	31.998	-.326
20.00	1.701	.011	3.145	2319.8	31.893	.012
21.00	1.911	.012	3.477	2464.8	32.506	-.017
22.00	2.137	.013	3.828	2616.2	33.333	-.347
23.00	2.380	.014	4.200	2774.7	33.466	-.266
24.00	2.639	.015	4.592	2941.3	33.439	.087
25.00	2.916	.016	5.006	3116.7	34.087	-.096
26.00	3.212	.018	5.442	3300.8	34.637	.395
27.00	3.527	.019	5.903	3491.9	34.564	-.066
28.00	3.863	.020	6.388	3675.6	34.799	.109
29.00	4.217	.022	6.894	3830.6	35.444	-.348
30.00	4.585	.023	7.417	3945.8	35.478	.155
31.00	4.966	.024	7.951	4019.8	35.631	.149
32.00	5.354	.025	8.490	4065.7	36.208	-.429
33.00	5.748	.027	9.033	4097.5	36.127	.145



## 32 LB PAYLOAD NOVEMBER 15, 1962

TIME SEC	THRUST LBS	WEIGHT LBS	TOTAL ACCEL. ACCEL./GRAV.	DYNAMIC PRES LBS/SQFT	DRAG LBS	MACH NO.
1.00	57010.29	13374.96	3.301	10.860	15.80	.09
2.00	54547.76	13127.38	3.191	47.322	69.65	.18
3.00	52610.84	12988.62	3.119	106.749	158.86	.27
4.00	51698.19	12855.75	3.132	197.785	282.50	.36
5.00	51808.49	12424.83	3.234	291.192	442.87	.45
6.00	51940.57	12193.81	3.320	417.893	652.68	.55
7.00	52510.28	11958.09	3.416	568.512	925.92	.65
8.00	53100.62	11722.28	3.542	743.312	1262.32	.75
9.00	53710.50	11486.46	3.689	940.122	2250.01	.85
10.00	54338.49	11250.64	3.820	1153.822	3866.83	.96
11.00	54982.73	11014.82	3.911	1377.406	5700.71	1.07
12.00	55641.17	10779.11	3.995	1609.652	6704.93	1.18
13.00	55895.34	10538.29	3.754	1851.356	7367.34	1.30
14.00	56160.19	10297.57	3.840	2098.764	7948.46	1.41
15.00	56433.88	10056.85	3.941	2349.222	8467.90	1.54
16.00	56714.54	9816.14	4.030	2599.334	8978.92	1.66
17.00	56750.29	9577.38	4.116	2844.754	9382.58	1.80
18.00	56789.15	9338.62	4.230	3081.406	9662.57	1.94
19.00	56829.16	9099.86	4.336	3306.185	9960.20	2.08
20.00	56868.37	8861.11	4.441	3514.101	10274.97	2.23
21.00	57404.90	8621.37	4.631	3704.676	10485.92	2.39
22.00	57937.04	8381.63	4.846	3877.729	10589.95	2.56
23.00	58463.16	8141.89	5.078	4030.255	10577.93	2.74
24.00	58981.72	7902.16	5.337	4159.142	10441.49	2.93
25.00	59491.21	7662.42	5.610	4260.411	10384.53	3.14
26.00	59990.10	7422.68	5.897	4328.212	10326.63	3.36
27.00	59226.90	7181.96	6.039	4352.484	10144.75	3.60
28.00	52947.41	6955.95	5.442	4199.541	9600.42	3.80
29.00	42980.17	6802.46	4.243	3936.209	8853.73	3.96
30.00	32990.77	6648.96	3.801	3587.477	7998.37	4.08
31.00	22979.30	6495.47	1.733	3198.064	7076.16	4.15
32.00	19113.52	6417.44	1.343	2787.837	6170.60	4.20
33.00	15228.90	6339.42	.945	2418.119	5341.95	4.23

32 LB PAYLOAD				NOVEMBER 15, 1962			
TIME SEC	X FEET	Y FEET	Z FEET	X-DOT FT/SEC	Y-DOT FT/SEC	Z-DOT FT/SEC	
1.00			10				
1.00	14		50	31		91	
2.00	61		190	64		190	
3.00	141		428	97		285	
4.00	258		760	136		378	
5.00	416		1184	182		470	
6.00	624		1698	236		560	
7.00	888		2306	291		654	
8.00	1207		3007	347		750	
9.00	1585		3804	410		847	
10.00	2026		4702	473		945	
11.00	2532		5695	539		1041	
12.00	3103		6784	605		1137	
13.00	3744		7970	675		1235	
14.00	4455		9254	748		1333	
15.00	5239		10637	822		1433	
16.00	6100		12121	900		1535	
17.00	7040		13708	980		1639	
18.00	8061		15399	1063		1744	
19.00	9166		17198	1148		1853	
20.00	10359		19105	1237		1962	
21.00	11642		21124	1329		2076	
22.00	13019		23254	1426		2194	
23.00	14494		25513	1527		2317	
24.00	16074		27994	1633		2446	
25.00	17763		30407	1746		2582	
26.00	19568		33059	1864		2724	
27.00	21493		35857	1987		2872	
28.00	23540		38804	2107		3012	
29.00	25700		41874	2210		3129	
30.00	27953		45047	2292		3212	
31.00	30276		48284	2350		3262	
32.00	32647		51562	2391		3288	
33.00	35056		54859	2425		3303	

32 LB PAYLOAD NOVEMBER 15, 1962

## ORBITAL ELEMENTS

SEMI-MAJOR AXIS	2.96904765E 003 N.M.
SEMI-MINOR AXIS	2.43921400E 003 N.M.
ECCENTRICITY	5.70147890E-001
PERIOD	6.76994746E 001 MIN.
APOGEE ALTITUDE	1.21794981E 003 N.M.
PERIGEE ALTITUDE	-2.16766551E 003 N.M.
INCLINATION	3.05078462E 001 DEG.
ARGUMENT OF PERIGEE	-2.63841916E 001 DEG.
LONGITUDE OF ASCENDING NODE AT BURNOUT TIME	1.65357821E 002 DEG.
INITIAL TRUE ANOMALY	1.39464460E 002 DEG.
INITIAL ECCENTRIC ANOMALY	1.09577040E 002 DEG.

NOVEMBER 15, 1962

32 LB PAYLOAD

## REPLETIAN TRAJECTORY

M	S	ALTITUDE(NM)	LATITUDE	LONGITUDE	INERTIAL VEL.(FPS)	INERTIAL FLY PATH ANG	RANGE(NM)
2	33	9.48868E 001	27.99566	-78.32510	23020.94	33.17988	123.29
3		1.50134E 002	27.67597	-76.93274	22578.52	32.83687	199.63
3	30	2.09502E 002	27.31510	-75.45445	22108.63	32.42341	281.35
4		2.66962E 002	26.94838	-74.03712	21658.91	31.97580	360.30
4	30	3.22538E 002	26.57684	-72.67558	21228.33	31.49504	436.68
5		3.76253E 002	26.20134	-71.36589	20815.97	30.98201	510.70
5	30	4.28131E 002	25.82260	-70.18429	20421.88	30.43750	582.52
6		4.78195E 002	25.44122	-68.88732	20042.65	29.86222	652.30
6	30	5.26467E 002	25.05770	-67.71192	19680.26	29.25688	728.20
7		5.72966E 002	24.67244	-66.57440	19333.20	28.62182	786.34
7	30	6.17715E 002	24.28578	-65.47300	19000.93	27.95781	850.85
8		6.60733E 002	23.89800	-64.40638	18682.95	27.26530	913.84
8	30	7.02038E 002	23.50930	-63.37006	18378.81	26.54475	975.42
9		7.41649E 002	23.11987	-62.36288	18088.12	25.79667	1035.69
9	30	7.79583E 002	22.72984	-61.38208	17810.93	25.02151	1094.73
10		8.15856E 002	22.33930	-60.42829	17545.70	24.21978	1152.64
10	30	8.50484E 002	21.94833	-59.49743	17293.36	23.39199	1209.48
11		8.83482E 002	21.55696	-58.58874	17053.26	22.53864	1265.33
11	30	9.14864E 002	21.16583	-57.70070	16825.17	21.66831	1320.26
12		9.44643E 002	20.77313	-56.83220	16608.90	20.75760	1374.34
12	30	9.72833E 002	20.38065	-55.98171	16404.28	19.83114	1427.63
13		9.99444E 002	19.98775	-55.14813	16211.17	18.88163	1480.18
13	30	1.02449E 003	19.59440	-54.33032	16029.42	17.90985	1532.05
14		1.04797E 003	19.20054	-53.52723	15858.94	16.91657	1583.29
14	30	1.06992E 003	18.80610	-52.73706	15699.63	15.90267	1633.95
15		1.09032E 003	18.41101	-51.96125	15551.42	14.86909	1684.07
15	30	1.10919E 003	18.01510	-51.19650	15414.22	13.81683	1733.72
16		1.12654E 003	17.61852	-50.44275	15288.00	12.74697	1782.91
16	30	1.14238E 003	17.22093	-49.69017	15172.70	11.66065	1831.71
17		1.15670E 003	16.82230	-48.96498	15068.29	10.55908	1880.14
17	30	1.16958E 003	16.42253	-48.23942	14974.75	9.44354	1929.26

32 LB PAYLOAD NOVEMBER 15, 1962

STATION - MOONERA

-31.380 N.LAT 136.890 E.LONG

FT.

TIME		TRAJECTORY (GEOCENTRIC)			LOOK ANGLES		SLANT		ASPECT
		ALTITUDE	LATITUDE	LONGITUDE	AZIMUTH	ELEVATION	RANGE	ANGLE	
M	S	N.MILES	DEG	DEG	DEG	DEG	NM	DEG	
2	33	95	27.84	-78.32	87.796	-74.444	6728	44.33	
3		150	27.52	-76.93	89.065	-74.884	6799	43.78	
3	30	210	27.17	-75.45	90.528	-75.357	6875	43.17	
4		267	26.80	-74.04	92.055	-75.817	6948	42.58	
4	30	323	26.44	-72.68	93.651	-76.262	7017	41.99	
5		376	26.06	-71.37	95.319	-76.691	7084	41.41	
5	30	428	25.69	-70.10	97.065	-77.105	7147	40.83	
6		478	25.31	-68.89	98.893	-77.503	7209	40.27	
6	30	526	24.93	-67.71	100.806	-77.895	7267	39.71	
7		573	24.55	-66.57	102.809	-78.249	7323	39.16	
7	30	618	24.16	-65.47	104.906	-78.596	7376	38.61	
8		661	23.78	-64.41	107.099	-78.925	7427	38.07	
8	30	702	23.39	-63.37	109.390	-79.235	7475	37.54	
9		742	23.01	-62.36	111.782	-79.525	7521	37.01	
9	30	780	22.62	-61.38	114.273	-79.795	7565	36.49	
10		816	22.23	-60.43	116.863	-80.043	7607	35.98	
10	30	850	21.84	-59.50	119.550	-80.270	7646	35.46	
11		883	21.45	-58.59	122.328	-80.473	7683	34.96	
11	30	915	21.06	-57.70	125.191	-80.654	7718	34.45	
12		945	20.67	-56.83	128.131	-80.811	7751	33.95	
12	30	973	20.28	-55.98	131.137	-80.943	7782	33.46	
13		999	19.89	-55.15	134.198	-81.050	7811	32.97	
13	30	1024	19.50	-54.33	137.299	-81.132	7837	32.48	
14		1048	19.11	-53.53	140.425	-81.189	7862	32.00	
14	30	1070	18.72	-52.74	143.561	-81.220	7885	31.52	
15		1090	18.32	-51.96	146.690	-81.227	7905	31.04	
15	30	1109	17.93	-51.20	149.797	-81.209	7924	30.57	
16		1127	17.54	-50.44	152.866	-81.168	7941	30.09	
16	30	1142	17.14	-49.70	155.883	-81.103	7956	29.63	
17		1157	16.74	-48.96	158.837	-81.015	7969	29.16	
17	30	1170	16.34	-48.24	161.719	-80.907	7981	28.69	

MODIFIED SLV-1C SWITS AUG 21, 1962 255 LRS					
STAGE 3					
TIME	LATITUDE	LONGITUDE	ALTITUDE	RANGE	
SEC	DEG	DEG	NM	NM	
1506.00	37.9374	214.6365	1111.19	1023.90	
1516.00	37.9527	214.4926	1098.42	1030.77	
1526.00	37.9681	214.3482	1085.74	1037.67	
1536.00	37.9834	214.2031	1072.58	1044.60	
1546.00	37.9988	214.0573	1059.11	1051.56	
1556.00	38.0141	217.9109	1045.34	1058.55	
1566.00	38.0295	217.7638	1031.27	1065.57	
1576.00	38.0448	217.6159	1016.49	1072.63	
1586.00	38.0601	217.4674	1002.21	1079.71	
1596.00	38.0755	217.3180	987.22	1086.83	
1606.00	38.0908	217.1679	971.93	1093.99	
1616.00	38.1062	217.0170	956.12	1101.18	
1626.00	38.1216	216.8653	940.39	1108.40	
1636.00	38.1370	216.7127	924.15	1115.67	
1646.00	38.1523	216.5592	907.59	1122.98	
1656.00	38.1677	216.4048	890.71	1130.32	
1666.00	38.1831	216.2494	873.50	1137.71	
1676.00	38.1986	216.0932	855.97	1145.14	
1686.00	38.2140	215.9359	838.11	1152.62	
1696.00	38.2294	215.7775	819.92	1160.14	
1706.00	38.2449	215.6182	801.39	1167.71	
1716.00	38.2604	215.4577	782.43	1175.34	
1726.00	38.2758	215.2961	763.32	1183.01	
1736.00	38.2914	215.1334	743.77	1190.73	
1746.00	38.3069	214.9695	723.48	1198.51	
1756.00	38.3224	214.8043	703.44	1206.34	
1766.00	38.3380	214.6378	683.04	1214.23	
1776.00	38.3536	214.4701	662.08	1222.18	
1786.00	38.3692	214.3010	640.77	1230.20	
1796.00	38.3848	214.1304	619.09	1238.27	
1806.00	38.4004	213.9585	597.05	1246.42	
1816.00	38.4161	213.7850	574.63	1254.63	
1826.00	38.4318	213.6100	551.44	1262.91	
1836.00	38.4475	213.4334	528.67	1271.26	
1846.00	38.4633	213.2552	505.11	1279.69	

MODIFIED SLV-1C SWTS AUG 21, 1962 255 LRS					
TIME SEC	VELOCITY FPS	FPA		AZIMUTH	
		DEG	DEG	DEG	DEG
1506.00	9357.2	-53.82		-A2.30	
1516.00	9505.9	-54.42		-A2.33	
1526.00	9656.6	-55.01		-A2.37	
1536.00	9809.3	-55.59		-A2.40	
1546.00	9963.9	-56.14		-A2.43	
1556.00	10120.4	-56.69		-A2.47	
1566.00	10279.0	-57.21		-A2.50	
1576.00	10439.4	-57.73		-A2.53	
1586.00	10601.9	-58.23		-A2.57	
1596.00	10766.3	-58.72		-A2.60	
1606.00	10932.7	-59.19		-A2.64	
1616.00	11101.1	-59.66		-A2.67	
1626.00	11271.5	-60.11		-A2.71	
1636.00	11444.0	-60.55		-A2.74	
1646.00	11618.5	-60.98		-A2.78	
1656.00	11795.1	-61.40		-A2.81	
1666.00	11973.8	-61.81		-A2.85	
1676.00	12154.6	-62.20		-A2.89	
1686.00	12337.6	-62.59		-A2.93	
1696.00	12522.8	-62.97		-A2.96	
1706.00	12710.3	-63.34		-A3.00	
1716.00	12900.0	-63.70		-A3.04	
1726.00	13092.0	-64.06		-A3.08	
1736.00	13286.4	-64.40		-A3.12	
1746.00	13483.2	-64.74		-A3.16	
1756.00	13682.5	-65.07		-A3.20	
1766.00	13884.3	-65.39		-A3.25	
1776.00	14088.7	-65.70		-A3.29	
1786.00	14295.7	-66.01		-A3.33	
1796.00	14505.4	-66.31		-A3.38	
1806.00	14717.9	-66.60		-A3.42	
1816.00	14933.2	-66.89		-A3.47	
1826.00	15151.4	-67.17		-A3.52	
1836.00	15372.6	-67.44		-A3.56	
1846.00	15596.9	-67.71		-A3.61	

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<p>system. All input and output data are in geodetic coordinates.</p> <p>Coasting flight trajectories are computed in two subroutines. The first is a Keplerian solution, which also computes orbital elements and "look angles" for various tracking stations. The second uses three-degree-of-freedom point mass equations solved by numerical integrations.</p> <p>The program will prepare two special output tapes. One is used in plotting output data and the other is used to prepare a special tape for the Atlantic Missile Range.</p>		<p>system. All input and output data are in geodetic coordinates.</p> <p>Coasting flight trajectories are computed in two subroutines. The first is a Keplerian solution, which also computes orbital elements and "look angles" for various tracking stations. The second uses three-degree-of-freedom point mass equations solved by numerical integrations.</p> <p>The program will prepare two special output tapes. One is used in plotting output data and the other is used to prepare a special tape for the Atlantic Missile Range.</p>	
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